Math 339 - Dynamical Systems
Assignment # 5
due Fri Nov 10th, 12:30pm

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. In class, we studied the Lotka-Volterra predator-prey system

$$\dot{x} = ax - bxy \tag{1a}$$
$$\dot{y} = \alpha bxy - dy. \tag{1b}$$

This system has two steady states: extinction at $(0,0)$ and coexistence at $(d/(\alpha b), a/b)$. By solving the phase plane equations, we found a Lyapunov function for the coexistence steady state:

$$\mathcal{L}(x,y) = a\ln(y) - by - \alpha bx - d\ln(x) + K'. \tag{2}$$

(a) Find the value of K' for which (2) is a Lyapunov function for (1).

(b) The function (2) is very different from the ones that we saw in class. If we were to propose a Lyapunov function similar to the ones seen in class, we would have tried

$$\mathcal{L}_1(x,y) = \left(x - \frac{d}{\alpha b}\right)^2 + \left(y - \frac{a}{b}\right)^2.$$

Show that $\mathcal{L}_1(x,y)$ is not a Lyapunov function for the coexistence steady state.

2. Consider the ODE system

$$\frac{dx}{dt} = ayx - x^2, \tag{3a}$$
$$\frac{dy}{dt} = \frac{ay^2}{x} - y, \qquad a > 0. \tag{3b}$$

(a) Write the phase plane equation and solve it.

(b) Sketch a few solutions in the phase plane.

3. Consider the system

$$\frac{dx}{dt} = x - 3y - 4 \tag{4}$$
$$\frac{dy}{dt} = x^2 + 4y. \tag{5}$$

Find all equilibria and classify them. Verify your results by producing a phase plane plot with Maple or pplane.