Math 339 - Dynamical Systems
Assignment # 6
due Wed Nov 29th, 12:30pm

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Consider the May predator-prey model (also called the Leslie-Gower model or the May-Holling-Tanner model):

$$\dot{n} = n(1-n) - \alpha \frac{n}{1+n} p, \tag{1a}$$

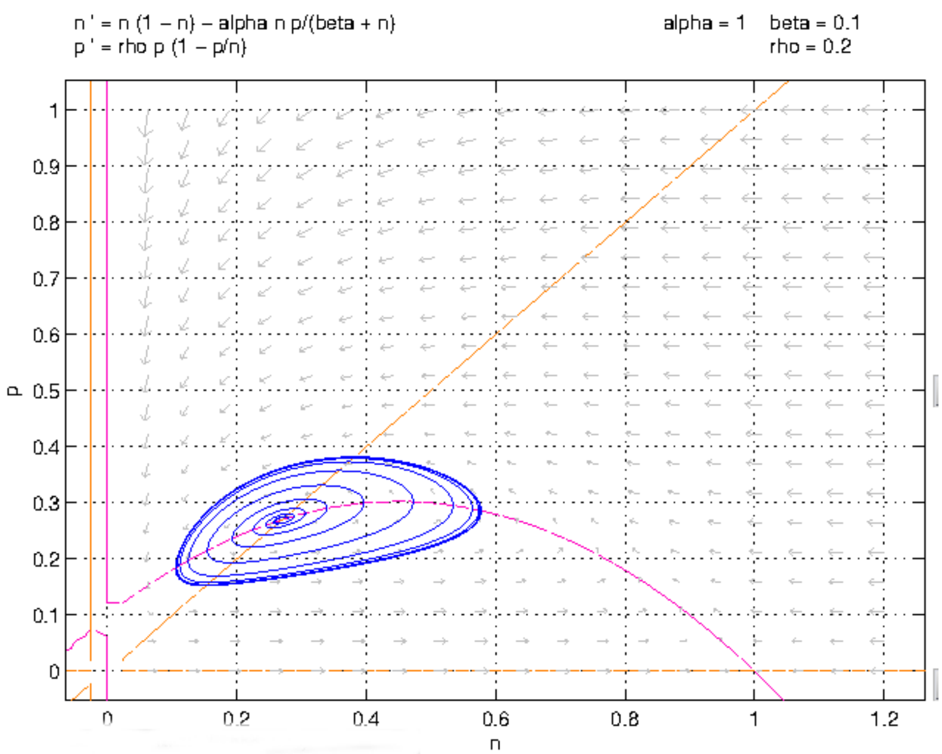
$$\dot{p} = \rho\, p \left(1 - \frac{p}{n}\right). \tag{1b}$$

We will focus on the case where $\alpha = 1$, $\rho = 0.2$, and $\beta = 0.1$.

(a) Find the steady states and their stability.

(b) For the given parameter values, the system has the limit cycle solution shown in Figure 1. Prove the existence of the limit cycle using the Poincaré-Bendixson Theorem. Use Figure 1 to help you figure out what trapping region you should use.

Figure 1: Figure for question # 1.



2. Plant-mycorrizhal associations are important mutualisms in the plant/fungus world. The following system of equations describes the mutualistic dynamics of a plant, p, and mycorrhizal fungus, m:

$$\dot{p} = \left(2m - \frac{m}{b+p} - p\right)p, \tag{2a}$$

$$\dot{m} = \left(2\frac{p}{b+p} - p - m\right)m, \tag{2b}$$

where $b > 0$ is an unknown parameter. Phase plane diagrams of (2) (made with pplane7) for $b = 0.5$ to 1.3 are shown in Figure 2. The blue curves are solution trajectories, while the pink and orange curves are nullclines.

Using the information in these diagrams, sketch the bifurcation plot for (2) using b as the bifurcation parameter and p^* (the $p-$coordinate of the steady state) on the vertical axis.

Figure 2: Figure for question # 2.