UBC ID #: ____________________ NAME (print): ____

Signature: ____

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 339
Date: Oct 4th, 2017 Time: 12:30pm Duration: 50 minutes.
This exam has 4 questions for a total of 40 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 50 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 30 minutes.

$\boxed{12}$ 1. In this question you will investigate the behaviour of solutions near the steady state. The plots you need are found on the next page.

(a) Let $f(x) = x - x^2$.

i. Show that $x = 0$ is a fixed point of f.

ii. Find $f'(x)$ at the fixed point. What do you conclude?

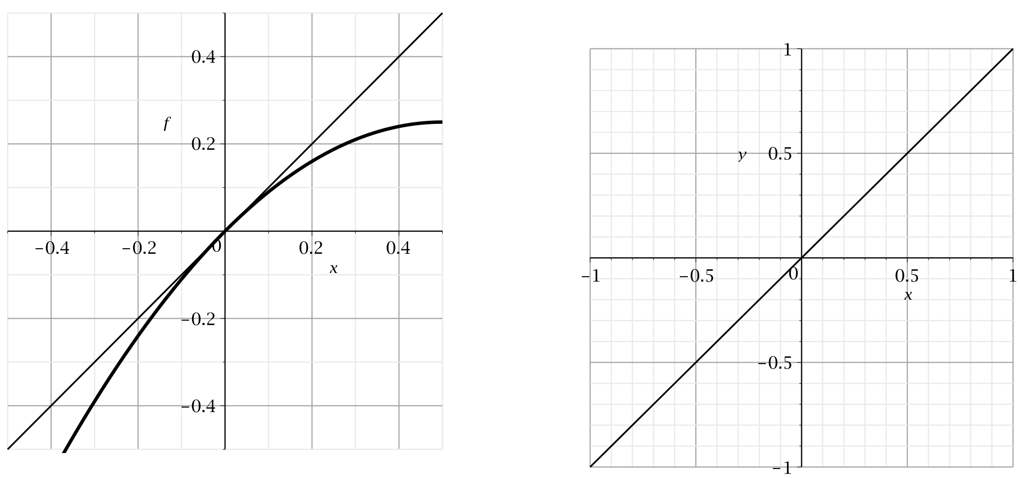
iii. Using the plot of $f(x)$ in Figure 1a, graphically determine the stability of the fixed point.

(b) Let $h(x)$ be an unknown function. Suppose that $h(x)$ satisfies $h'(0) = -1$ and $x = 0$ is an attracting fixed point.

i. Sketch a possible $h(x)$ on the axes in Figure 1b. (Keep your function very simple!)

ii. Use your sketch to graphically demonstrate that the fixed point has the required stability property.

Figure 1: Figures for question # 1.



(a) Plot of $f(x)$ (thick curve) with the identity line (thin line)

(b) Plot of $h(x)$ with the identity line.

$\boxed{10}$ 2. Consider the map $h(x) = axe^{-x}$ where $a \in \mathbb{R}$, $a > 0$.

(a) Find the fixed points of the map $h(x)$.

(b) Find out for what value of a the positive steady state x^* is a superstable fixed point x^*, that is, $h'(x^*) = 0$. Denote by a_{ss} this value of a.

(c) What is the range of $a \geq 1$ for which the steady state x^* is a sink? You may find Figure 2 helpful.

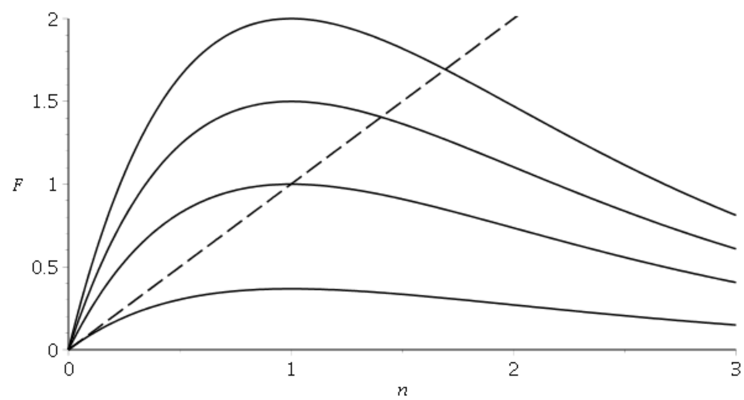


Figure 2: Plot of the identity line (dashed) with the map $h(x)$ for $a = 1$, e, $1.5e$, and $2e$ (solid curves). The plot of $h(x)$ becomes more peaked as a increases.

$\boxed{10}$ 3. Consider the map

$$f(x,y) = \left(\frac{x}{2}, 2y - \frac{7x^3}{8}\right)$$

The only fixed point of f is (0,0).

(a) Find the inverse map f^{-1}.

(b) Show that the set $S = \{(x, rx^3) : x \in \mathbb{R}\}$ is invariant under f.

(c) Show that each point in S converges to $(0,0)$ under f.

$\boxed{8}$ 4. Consider the bifurcation plot in Figure 3.

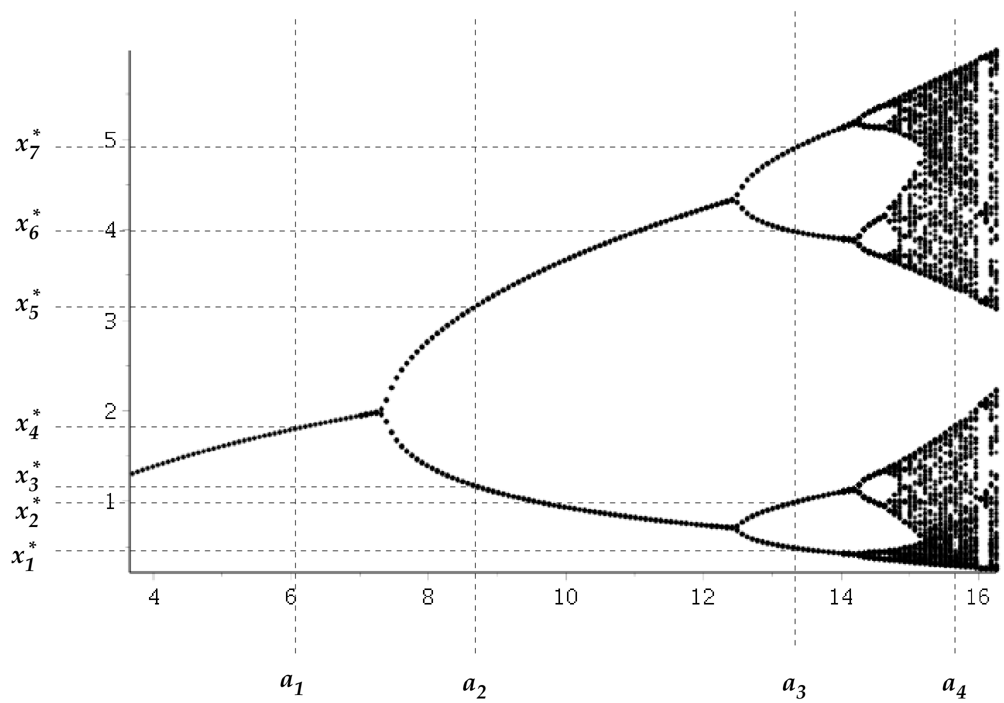


Figure 3: Bifurcation plot for the map $g(x) = axe^{-x}$.

(a) What would the time series' of $g(x)$ orbits look like for $a = a_1$, a_2, and a_3? Make a sketch for each one. Make sure that your time series is long enough so that the behaviour is clearly identifiable. Assume that transients have passed and you are only plotting the steady state behaviour. *(Hint: A time series is a plot of x_n vs n.)*

(b) What is the behaviour at a_4 called? Name three characteristics of this behaviour.

Question:	1	2	3	4	Total
Points:	12	10	10	8	40
Score:					