UBC ID #: ____________ NAME (print): ____________________

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a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

IRVING K. BARBER SCHOOL
OF ARTS AND SCIENCES
UBC OKANAGAN

Instructor: Rebecca Tyson Course: MATH 339
Date: Nov 17th, 2017 Time: 12:30pm Duration: 50 minutes.
This exam has 5 questions for a total of 45 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 50 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 30 minutes.

1. Consider the linear system

$$\frac{dx}{dt} = 3x - y, \tag{1a}$$
$$\frac{dy}{dt} = 2x + 4y. \tag{1b}$$

$\boxed{4}$ (a) Determine the stability of the steady state at the origin.

$\boxed{4}$ (b) Sketch the phase plane (use nullclines).

$\boxed{5}$ 2. Consider the system

$$\frac{dx}{dt} = -z + \left(\frac{x^3}{3} - x\right), \tag{2a}$$
$$\frac{dz}{dt} = x. \tag{2b}$$

Show that $V(x, z) = x^2 + z^2$ is a Lyapunov function for the (0,0) steady state of (2) over part of the (x, z) phase plane.

$\boxed{3}$ 3. What is a Hamiltonian system? What can you say about their Lyapunov functions and phase plane solution trajectories?

$\boxed{5}$ 4. Consider the nonlinear system

$$\frac{dx}{dt} = \rho y x - y^2, \tag{3a}$$
$$\frac{dy}{dt} = \frac{y}{x} - \rho, \qquad \rho > 0. \tag{3b}$$

Write the phase plane equation and solve it. What do the solutions of the phase plane equation represent?

5. Consider the competition system

$$\frac{dx}{dt} = x(4 - x) - 2xy, \tag{4a}$$
$$\frac{dy}{dt} = y(3 - y) - \alpha xy. \tag{4b}$$

$\boxed{4}$ (a) Assuming that we are only interested in solutions where $x \geq 0$ and $y \geq 0$, find the four steady states.

$\boxed{2}$ (b) Sketch the nullclines. Label all of the intercepts and intersection points.

$\boxed{10}$ (c) Determine the stability of each of the four steady states. The stability of two of the steady states will depend on the value of α. *Hint: The calculations for the coexistence steady state simplify if you do two things: (1) factor the common denominator out of the Jacobian and find the eigenvalues of the remaining matrix, and (2) before computing the eigenvalues, let $\eta = 4\alpha - 3$ and rewrite the matrix in terms of η instead of α.*

$\boxed{4}$ (d) Sketch the phase plane (two cases).

$\boxed{4}$ (e) Sketch the bifurcation diagram for the x-coordinate of each steady state (α is the bifurcation parameter). Since there are two steady states with $x^* = 0$, do not include the extinction steady state in your diagram.

Question:	1	2	3	4	5	Total
Points:	8	5	3	5	24	45
Score:						