# Math 339 - Dynamical Systems
# Sep-Dec 2019
# Assignment # 1

**Instructions:** You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Modify the opinion dynamics model to include stubborn agents. To simplify things, assume only two opinions: $-1$ and $+1$. There are thus four types of individuals:

$$\begin{aligned}
&X_{-1}, && \text{non-stubborn agents holding opinion } -1,\\
&X_{+1}, && \text{non-stubborn agents holding opinion } +1,\\
&S_{-1}, && \text{stubborn agents holding opinion } -1,\\
&S_{+1}, && \text{stubborn agents holding opinion } +1.
\end{aligned}$$

Assume that S agents do not change their opinion unless there are sufficient $X$ agents of the opposite opinion. That is, an $S_{-1}X_{+1}$ interaction results in the $S_{-1}$ individual becoming an $X_{-1}$ individual (no change of opinion, but no longer stubborn), if the $X_{+1}$ population is large enough. The switching rate is $f(X_{+1})$. Interactions between stubborn agents do not result in any changes (stubborn agents just yell at each other). Interactions between $X$ agents are as described for the model in class, except that there is no amplification. Instead of amplification, $X$ agents become stubborn if there are sufficient $S$ agents of the same opinion. The switching rate in this case is $f(S_i)$, where $i$ is the appropriate opinion type. For example, an $X_{+1}$ agent interacting with an $S_{+1}$ agent will become stubborn with probability $f(S_{+1})$. The transition diagram for population movement between subgroups is shown in Figure 1.

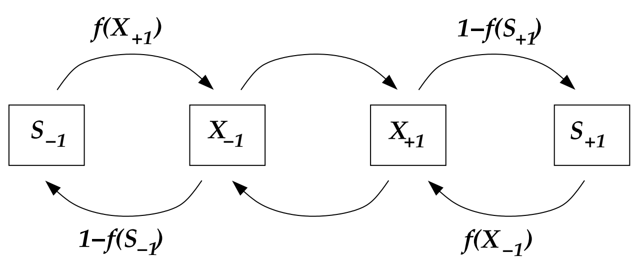


Figure 1: Transition diagram, with rates, for the movement of population subgroups.

(a) Propose a plausible function for $f(X_i)$. Note that it is a probability function, and must satisfy $f(0) = 0$ and $f(1) = 1$. Sketch the function.

(b) Write the four model equations.

2. Verify that equation (7b) in [1] is correct (the paper is available on the course web page under Lecture Notes). You may need to use the approach described in Appendix A of the paper. *Hint: You will need to use the second formulation discussed in class, that is,* $\dot{L}_1 =$ (*gain interactions*) $-$ ($L_1 -$ (*steady state interactions*)). *You will also need to use the fact that* $L_2 + L_1 + R_1 + R_2 = 1$.

3. For the nonlinear ODE systems below, find the nullcines and steady states, and sketch the phase plane.

$$
\begin{aligned}
&\text{(a)} \quad \dot{x} &&= x\sin(y),\ \text{(only consider } -7 < y < 7 \text{ (just 5 steady states))} \\
& \quad\quad\ \dot{y} &&= x^3 - y \\
&\text{(b)} \quad \dot{x} &&= 10xy - 1, \\
& \quad\quad\ \dot{y} &&= x + y - 1 \\
&\text{(c)} \quad \dot{x} &&= (y + 2x - 1)^2, \\
& \quad\quad\ \dot{y} &&= y - x^2
\end{aligned}
$$

Note: Additional questions will be added in the upcoming days!

## References


[1] B.O. Baumgaertner, P.A. Fetros, S.M. Krone, and R.C. Tyson (2018) Spatial opinion dynamics and the effects of two types of mixing, *Physical Review E* **98** 022310