# Math 339 - Dynamical Systems
# Sep-Dec 2019
# Assignment # 2

**Instructions:** You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. The nonlinear ODE systems below appeared in your first assignment. For each model, linearise about the indicated steady state, and determine its stability. Name the type of steady state.

   (a) $$\dot{x} = x\sin(y), \quad \dot{y} = x^3 - y, \quad (x^*, y^*) = (2\pi)^{1/3}, 2\pi),$$

   (b) $$\dot{x} = 10xy - 1, \quad \dot{y} = x + y - 1, \quad (x^*, y^*) = \left(\frac{5+\sqrt{15}}{10}, \frac{5-\sqrt{15}}{10}\right),$$

   (c) $$\dot{x} = (y + 2x - 1)^2, \quad \dot{y} = y - x^2, \quad (x^*, y^*) = (-1-\sqrt{2}, 3 + 2\sqrt{2})$$

2. In class, we considered the dynamical system

   $$\begin{aligned} \dot{x} &= 3x - y, \\ \dot{y} &= 6x - 4y, \end{aligned}$$

   and showed that the solutions are given by $\vec{x} = c_1\vec{v_1}\exp 2t + c_2\vec{v_2}\exp -3t$. We then showed that the line defined by $\vec{v_1}$ is an invariant set and unstable manifold for the dynamical system. Find the invariant set defined by $\vec{v_2}$ (your answer should include a proof that the set you have defined is indeed invariant). Does this set define a stable or unstable manifold?

3. For the simplified opinion dynamics model that we considered in class (equations (8) in [1], or equations(2) in lecture #2), show that the set $L = 0$ where $L = 1/2 - x - y$ is an invariant set for the system when $p_a = 0$. Is it the stable or unstable manifold?

## References

[1] B.O. Baumgaertner, P.A. Fetros, S.M. Krone, and R.C. Tyson (2018) Spatial opinion dynamics and the effects of two types of mixing, *Physical Review E* **98** 022310