Back to the seasonal predator-prey model: Case 1, alternative prey insufficient:

$$(1)\ldots\begin{cases} \dfrac{dx}{d\tau} = T_s\left[x(1-x) - \dfrac{x^2 y}{b^2+x^2}\right] + (1-T_s)\left(-\dfrac{\alpha x y}{\beta + x}\right) \\ \dfrac{dy}{d\tau} = T_s\left[\gamma \dfrac{x^2 y}{b^2+x^2} + (s-m)y\right] + (1-T_s)\left(\gamma \dfrac{\alpha x y}{\beta+x} - \mu y\right) \end{cases}$$

with $s < m$.

Parameter values: $\alpha = 2$, $\beta = 0.05$, $s = 0.5$, $m = \mu = 0.6$, $\gamma = 0.25$, $b = 0.25$, $0.4 < T_s < 0.9$

Steady-States:

$(0,0)$; $(1,0) \to$ prey only

$\hookrightarrow$ extinction

$(x^*, y^*) \to$ coexistence

Nullclines:

$$\begin{cases} f(x,y) = 0 \Leftrightarrow x = 0 \text{ or } y = \dfrac{T_s(1-x)(b^2+x^2)(\beta+x)}{T_s x(\beta+x) + (1-T_s)\alpha(b^2+x^2)} \\ g(x,y) = 0 \Leftrightarrow y = 0 \text{ or } T_s\left[\dfrac{\gamma x^2}{b^2+x^2} + (s-m)\right] + (1-T_s)\left(\dfrac{\gamma\alpha x}{\beta+x} - \mu\right) = 0 \end{cases}$$

Explore with pplane.

Load the system: Math339/2019/LectureNotes/lecture10_ghocasel.pps

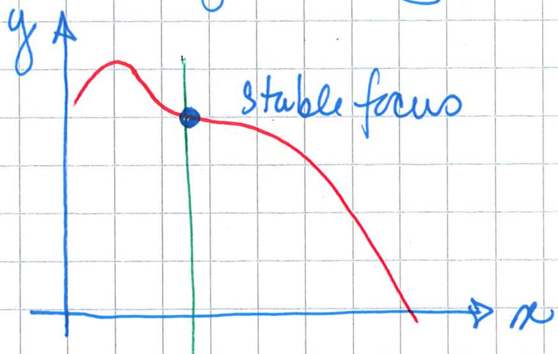
Set $T_S = 0.5$
$T_S = 0.55$
$T_S = 0.75$
$T_S = 0.9$

(can also look at $T_S = 0.8$)

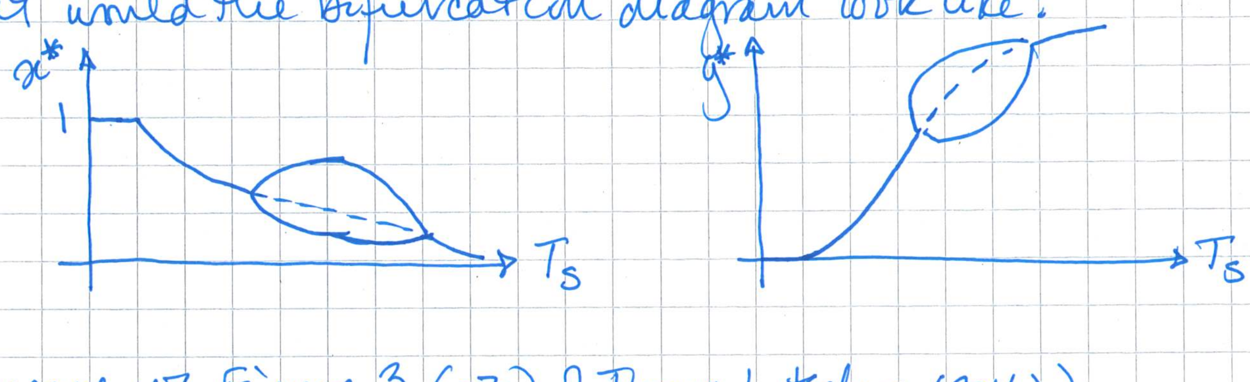
→ as T_S increases, this nullcline moves to the left
→ we expect a Hopf bifurcation at * b/c of this model's similarity to the model studied in Lecture 8 (Rosenzweig - MacArthur model)

What do we observe?

- stable node → stable focus → Hopf bifn + limit cycle
- red nullcline gradually changes shape, getting flatter at the top & eventually developing a second peak (transitioning to summer-dominated dynamics)



What would the bifurcation diagram look like?



(Compare w/ Figure 3 (p7) of Tyson & Lutscher (2016))

Now consider Case 2, Alternative prey sufficient

$$(2)\cdots\begin{cases} \dfrac{dx}{d\tau} = \text{same as in (1)} \\ \dfrac{dy}{d\tau} = T_S\left[\gamma\dfrac{x^2y}{b^2+x^2} + s\dfrac{y}{1+\nu y} - my\right] + (1-T_S)\left(\gamma\dfrac{\alpha x y}{\beta+x} - \mu y\right) \end{cases}$$

Steady-states:

$(0,0)$; $(1,0)$ → prey only

↳ extinction

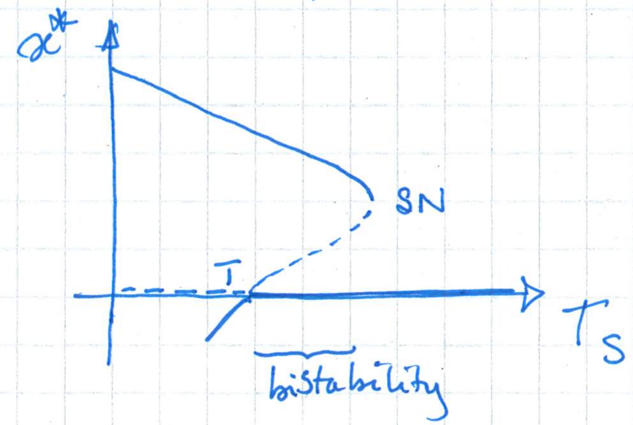
(x^*, y^*) → coexistence ...

$\left(0, \frac{1}{\nu}\left(\frac{m}{sT_S} - 1\right)\right)$ → predator only if T_S suff. large

Explore with pplane. Parameter Set #1.

Set $T_S = 0.57$
0.59
0.60
0.62

What would the bifurcation diagram look like?



Compare w/ Figure 5a (p9) of Tyson & Lutscher (2016).

Explore with pplane. Parameter Set #2.

Set $T_S = 0.39$
0.4
0.407
0.41
0.415
0.42
0.425

Complicated bifurcation diagram... (Compare w/ Figure 5b (p 9) of Tyson & Lutscher (2016).)