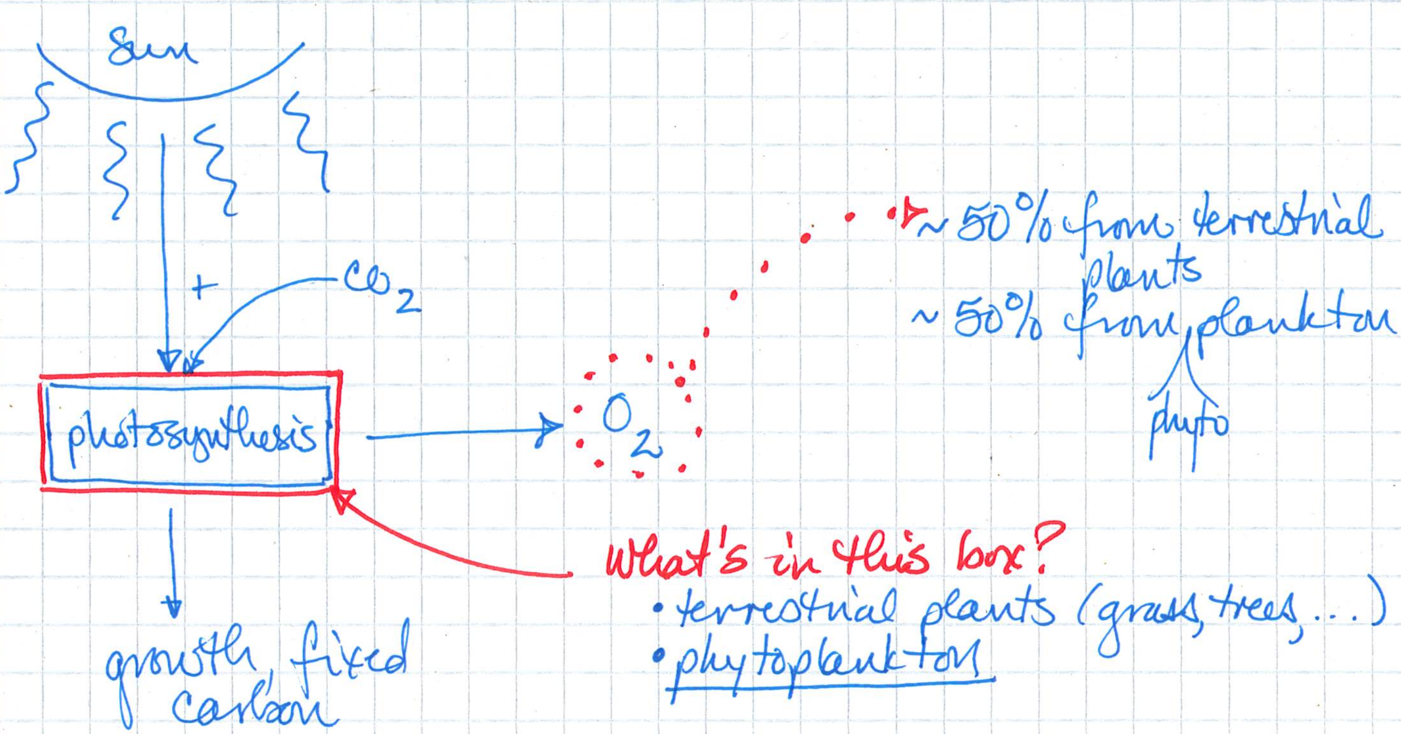
Lecture #11

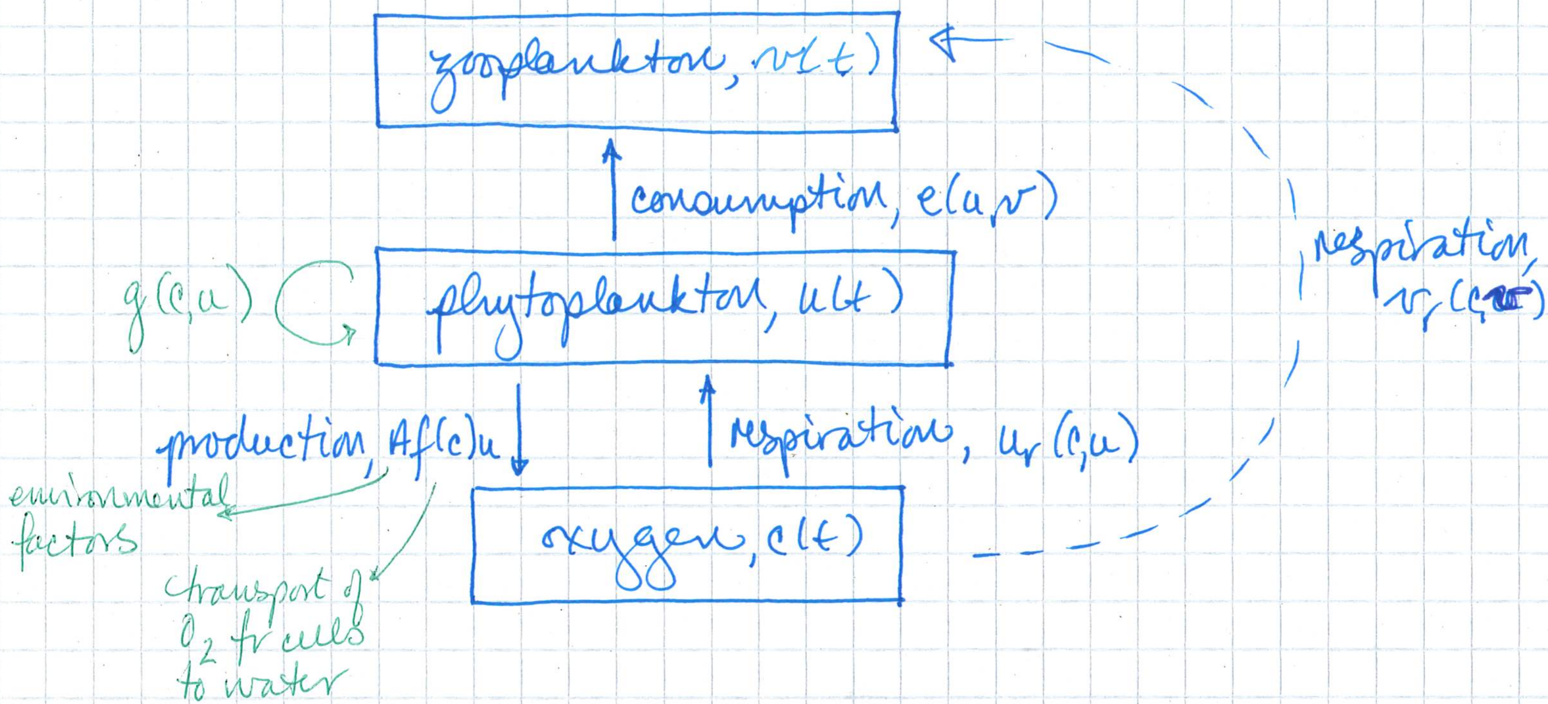
Modelling Plankton-Oxygen Dynamics under Climate Change



Phytoplankton primer
- live in the ocean
- are affected negatively by ocean temperature + acidity

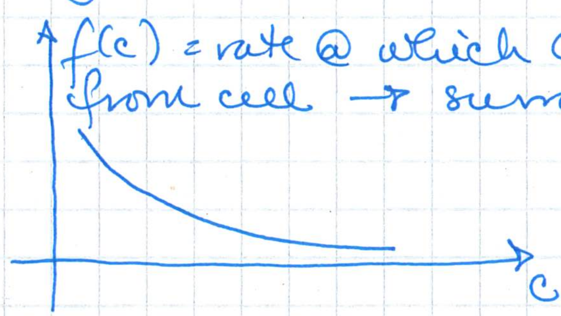
Paper:
1. Sekerci + Petrovskii (2018) Geosciences 8:201
2. Petrovskii, Sekerci, + Venturino (2017) JTB 424: 91-109
3. Sekerci + Petrovskii (2015) BMB 77:2325-2353

The Model (2015 paper)



$$
(1)\ldots\begin{cases}
\dfrac{dc}{dt} = Af(c)u - u_r(c,u) - v_r(c,v) - mc, \\[2mm]
\dfrac{du}{dt} = g(c,u) - e(u,v) - \sigma u, \\[2mm]
\dfrac{dv}{dt} = \underbrace{K(c)}_{\text{conversion efficiency}} e(u,v) - \mu v
\end{cases}
$$

Selecting specific, plausible fns: Example: $f(c)$



* (monotonically decreasing)

so let $f(c) = 1 - \dfrac{c}{c+c_0} = \dfrac{c_0}{c+c_0}$

Similar arguments lead to:

$$(2)\ldots\begin{cases}\dfrac{dc}{dt} = \dfrac{A c_0 u}{c+c_0} - \dfrac{\delta u c}{c+c_2} - \dfrac{\nu c v}{c+c_3} - mc\\[2mm] \dfrac{du}{dt} = \left(\dfrac{Bc}{c+c_1} - \gamma u\right)u - \dfrac{\beta u v}{u+h} - \sigma u\\[2mm] \dfrac{dv}{dt} = \dfrac{\eta c^2}{c^2+c_4^2}\ \dfrac{\beta u v}{u+h} - \mu v\end{cases}$$

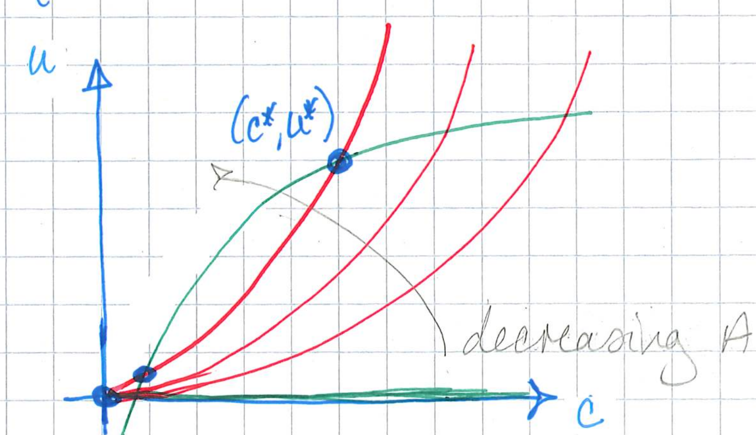
All parameters are nonnegative.

The dimensionless form of the model is

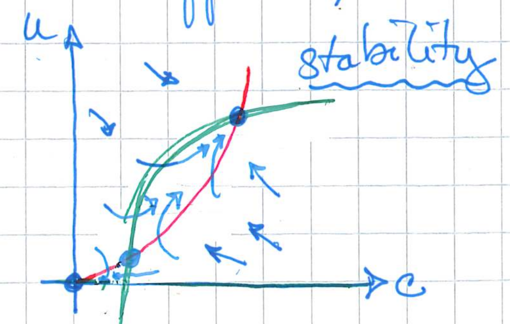
$$(3)\ldots\begin{cases}\dot{c} = \dfrac{Au}{c+1} - \dfrac{\delta u c}{c+c_2} - \dfrac{\nu c v}{c+c_3} - c\\[2mm] \dot{u} = \left(\dfrac{Bc}{c+c_1} - u\right)u - \dfrac{uv}{u+h} - \sigma u\\[2mm] \dot{v} = \dfrac{\eta c^2}{c^2+c_4^2}\ \dfrac{uv}{u+h} - \mu v\end{cases}$$

<u>Analysis: look @ just O_2 - phytoplankton (so $v(t)=0$).</u>

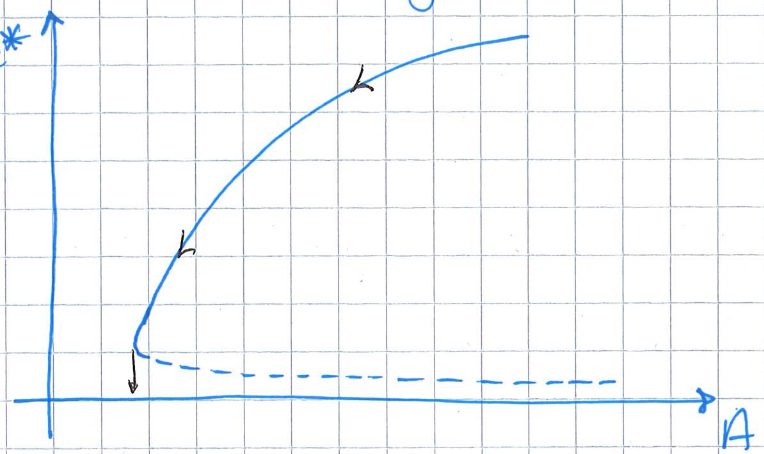
$$(4)\ldots\begin{cases}\dot{c} = \dfrac{Au}{c+1} - \dfrac{\delta u c}{c+c_2} - c & \Rightarrow \text{oxygen isocline}\\[2mm] \dot{u} = \left(\dfrac{Bc}{c+c_1} - u\right)u - \delta u & \Rightarrow \text{phytop. "}\end{cases}$$



A = bifn parameter (environmental effects)



Bifurcation Diagram



- saddle-node bifurcation
- critical transition to extinction
- recovery not possible

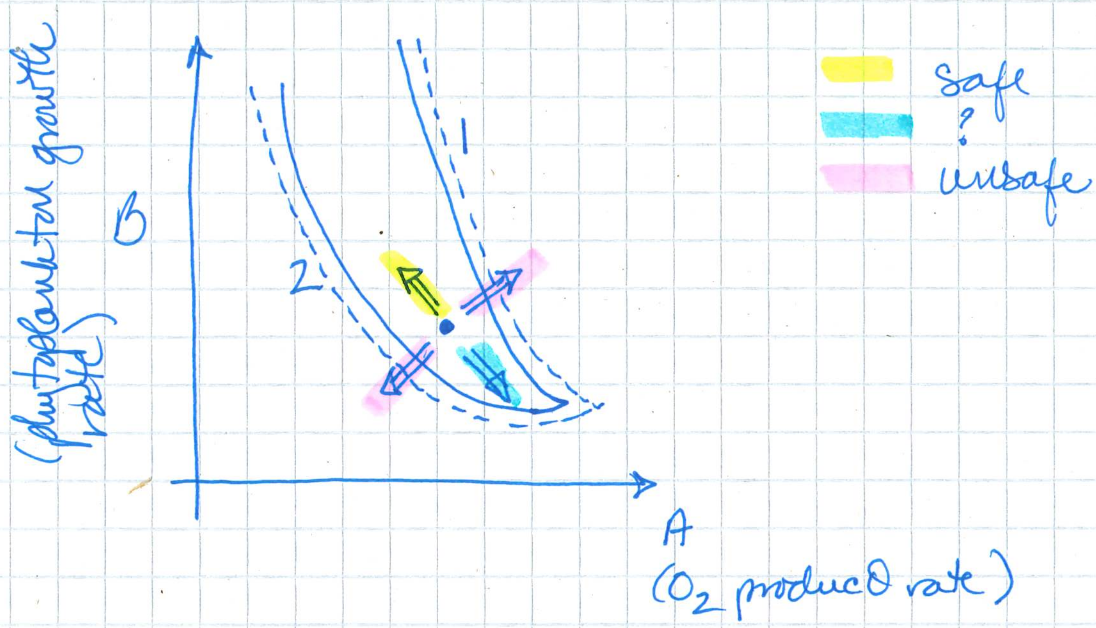
Now consider the full model

- The coexistence equilibrium exists if A is not too large or too small.
- When A increases, there is a Hopf bifurcation leading to a limit cycle. → c(t), u(t), + v(t) all periodic

→ numerical study (lots of simulations w/ lots of parameter values)

(2017 paper)

Two-parameter bifurcation diagram...



1) Fig 4d → e → f (catastrophe Type 1)
   - onset of spatial oscillations
   - amplitude increases
   - collapse

2) Fig 4d → c → b → a (catastrophe Type 2)
   - no oscillations precede the collapse