

Last time:

- studied an O_2 -phytoplankton model
- found a saddle-node bifurcation leading to global stability of extinction as O_2 productivity (A) decreases.
- briefly looked at the bifurcation diagram for the O_2 -phytoplankton-zooplankton model + found that extinction occurs (at or w/o a Hopf bifurcation) if A too large or too small.

Questions

- (1) How long is the time to extinction?
- (2) How does this prediction change if
 - (a) the food web is more complex?
 - (b) stochasticity is added?
- (3) Are there any early warning signals?

Answers

(2a) Comparison theorem for plankton models (Appendix A in the 2017 paper)

The fully general model: (28) - (33)

Bounding of (28): (34)

(27): (35)

(30): (36)

Grönwall's Inequality (notes from Wikipedia)

Upper bound model: (37) - (42)

⇒ the simple O_2 -phytoplankton-zooplankton model is an upper bound for a very large class of more general models

⇒ if extinction happens at time T in the simple model, then it will happen at time $t \leq T$ in the more complex models

⇒ see Figure 16 of the (2018) paper

$$(2b) \quad \begin{aligned} A(t, \xi) &= (A_0 - \omega_A t) + A_D \xi \\ c_i(t, \xi) &= (c_{i,0} - \omega_c t) + c_{i,D} \xi \end{aligned} \quad (\text{2018 paper})$$

$\xi \rightarrow$ a random variable (normally-distributed)
 $\omega_A + \omega_c \rightarrow$ steady change of the mean due to global warming
 $A_0 + c_{i,0} \rightarrow$ initial means
 $A_D + c_{i,D} \rightarrow$ noise intensity

Results:

- amplitude of oscillations increases with noise (Figs 10 + 11)
- time to extinction increases with noise (Figs 12 + 13)
- note that Fig 13 also shows the lower time to extinction for the more complex model

(3) (2017 paper) \rightarrow spatial model

- changes in the temporal variability (decrease) of the average oxygen concentration
- increase in the regularity of the spatial pattern

(1) ? We would need empirically-measured parameter values to know.

Remarks

- Why a modelling approach? (2017 paper, last para p1 - 1st para p2)
- What evidence do we have that the apocalyptic outcome could actually occur? (2018 paper, last para p18)