September 5

Thursday, September 5, 2019 3:28 PM

What is a dynamical system ?

↳ any system that evolves through time

i.e . height of tree as a function of time - one player system

- population of predators & prey - interactive system

  ↑ humans ↑ fish

  lynx hare

- pendulum
- traffic flow
- disease spread

Mathematically, a dynamical system is a set of

continuous { - ordinary differential equations
- partial " "

mixed → - integro " " (involves both integrals & differential)

discrete → - difference "

Goals

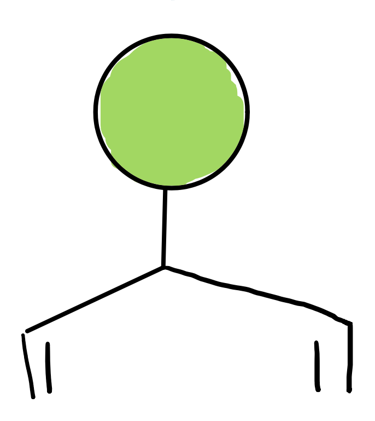
. ODEs : . intepret

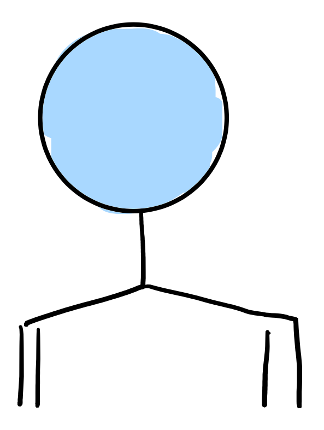
. solve → graphically / steady-state / numerically .

Model 1 - Opinion Dynamics

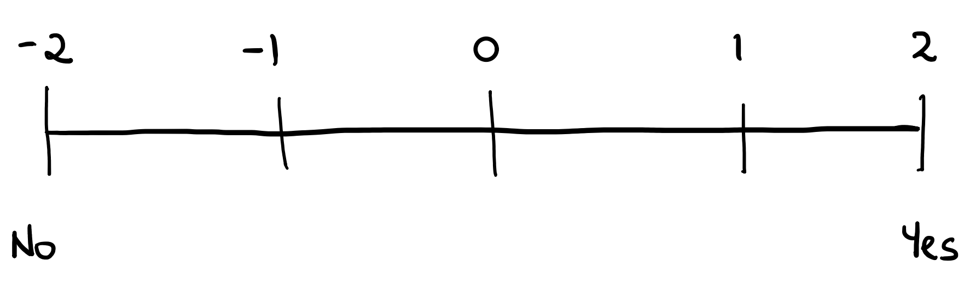
Types of Behaviours

- steady- state
- steady increase / decrease
- oscillation





How to measure opinion ? (single issues)



🟦 focal individual

🟩 neighbour

No Amplification

	-2	-1	1	2
-2	-2	-1	-1	-1
-1	-2	-1	1	1
1	-1	-1	1	2
2	1	1	1	2

Rule : The focal individual moves 1 step forward the opinion of the chosen neighbour

Amplication Case

(probability Pa)

	-2	-1	1	2
-2	-2	-2	-1	-1
-1	-2	-2	1	1
1	-1	-1	2	2
2	1	1	2	2

Rule : The focal individual's opinion becomes more entrenched (1step) if interacting w/ a neighbour of the same side.

Let :

L_2 = fraction of individuals with opinion -2

L_1 = " -1

R_1 = 1

R_2 = 2

summation = 1

$$\frac{dL_2}{dt} = \boxed{\text{interactions that increase } L_2} - \boxed{\text{interactions that decrease } L_2}$$

🟥 Increases L_2

🟪 decreases L_2

▨ no change

No Amplification

$(1-p_a)$

	-2	-1	1	2
-2	▨	🟪	🟪	🟪
-1	🟥			
1				
2				

Amplication Case

(probability Pa)

	-2	-1	1	2
-2	▨	▨	🟪	🟪
-1	🟥	🟥		
1				
2				

$$\frac{dL_2}{dt} = \left[(1-\cancel{p_a})L_1L_2 + \cancel{p_a L_1 L_2} + p_a L_1 L_1\right]$$

$$- \left[(1-p_a)L_2\left(L_1 + \cancel{R_1 + R_2}\right) + \cancel{p_a L_2 (R_1 + R_2)}\right]$$

$$= \left[L_1L_2 + p_a L_1 L_1\right] - \left[(1-p_a)L_2 L_1\right] \quad ☒$$

$$\frac{dL_1}{dt} = \boxed{\text{interactions that increase } L_1} - \boxed{\text{interactions that decrease } L_1}$$

🟥 Increases L_1

🟪 decreases L_1

▨ no change

No Amplification

$(1-p_a)$

	-2	-1	1	2
-2		🟥	🟥	🟥
-1	🟪	▨	🟪	🟪
1	🟥	🟥		
2				

Amplication Case

(probability Pa)

	-2	-1	1	2
-2			🟥	🟥
-1	🟪	🟪	🟪	🟪
1	🟥	🟥		
2				

$$\frac{dL_1}{dt} = \Big[(1-p_a)(L_1L_2 + R_1L_2 + R_2L_2) + p_a(R_1L_2 + R_2L_2)$$

$$+ (1-p_a)R_1(L_2 + L_1) + p_a R_1 (L_2 + L_1)\Big]$$

$$- \Big[(1-p_a)L_1(L_2 + R_1 + R_2) + p_a L_1(L_2 + L_1 + R_1 + R_2)\Big]$$

↳ check literature paper : answer on pg 4

Another way to model the change in L_2 :

$\dot{L}_2$ = increased - decreased

= increased - [L_2 - no change]

. Read Appendix A