September 10

Tuesday, September 17, 2019 5:03 PM

Homework Notes : switching rate : $f(X_{t+1})$ is a function $\therefore$ $0 \leq f(X_{t+1}) \leq 1$

↳ switching rate can be a constant (example: pa like previous week)

Opinion Dynamics Model

$$
\left.
\begin{aligned}
\dot{L}_2 &= L_1[L_2 + p_a L_1] - L_2[1 - L_2 - p_a L_1] \\
\dot{L}_1 &= L_2[1 - L_2 - p_a L_1] + R_1[L_2 + L_1] - L_1[1 - (1-p_a)L_1] \\
\dot{R}_1 &= R_2[1 - R_2 - p_a R_1] + L_1[R_1 + R_2] - R_1[1 - (1-p_a)R_1] \\
\dot{R}_2 &= R_1[R_2 + p_a R_1] - R_2[1 - R_2 - p_a R_1]
\end{aligned}
\right\} \quad \ldots\ (1)
$$

Observation

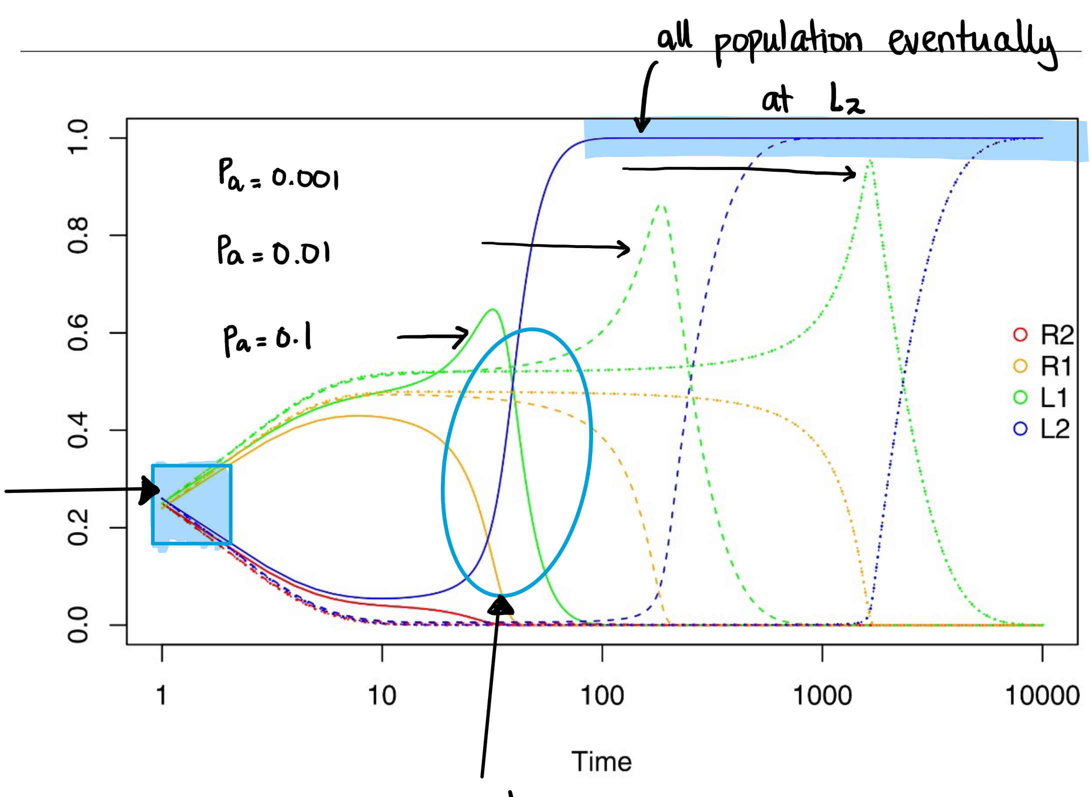
- consensus : everyone is L_2 eventually (since L_2 was largest initially)
  - ↳ would be R_2 if initial conditions gave the R side a small advantage
- rapid movement to consensus after initial period
- centering : initially, R_1 & L_1 ↑ while R_2 & L_2 ↓
- quasi steady-state between centering & consensus
  - ↳ time period ↑ as pa ↓
- initial behaviour is the same
  - ↳ as amplification (pa) ↑ , the faster the population reaches consensus.

pa = 0.1 reaches consensus @ 100

pa = 0.01 " " @ 1000

pa = 0.001 " " @ 10 000

initial behaviour is the same



rapid movement to consensus after initial period (steep slopes)

Simulation

- **Step 1 : Centering**
  - $L_1 = R_1 = x$
  - $L_2 = R_2 = y$
- **Step 2 : Consensus**
  - $R_1 = R_2 = 0$
  - $L_1 = x$
  - $L_2 = y$

} Proposed

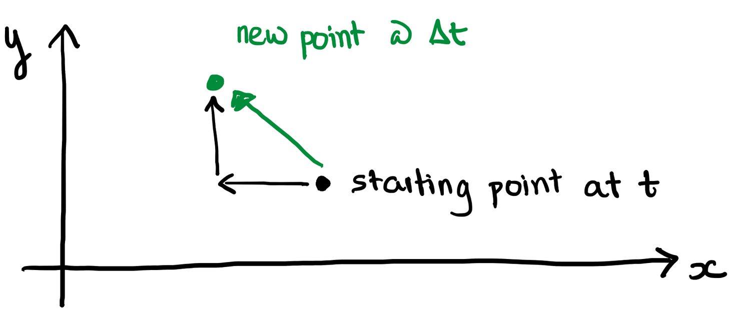
Step 1 Simplified model for centering

$$
\left.
\begin{aligned}
\dot{y} &= x[y + p_a x] - y[1 - y - p_a x] \\
\dot{x} &= y[1 - y - p_a x] + x[y + x] - x[1 - (1-p_a)x]
\end{aligned}
\right\} \quad \ldots\ (2)
$$

How do we analyze ?

↳ autonomous ODEs (time does not appear)

Phase Plane Analysis



If we fill the (x,y) plane w/ little arrows ⇒ tell us how system (2) moves in phase plane as time increases

Analysis of a Non-Linear ODE System

1) Nullclines + Steady States

Nullclines ↓ curves along which either $\dot{x} = 0$ or $\dot{y} = 0$

Steady States ↓ (equilibrium points) points at which the system is "steady" (or unchanging) $\dot{x} = \dot{y} = 0$

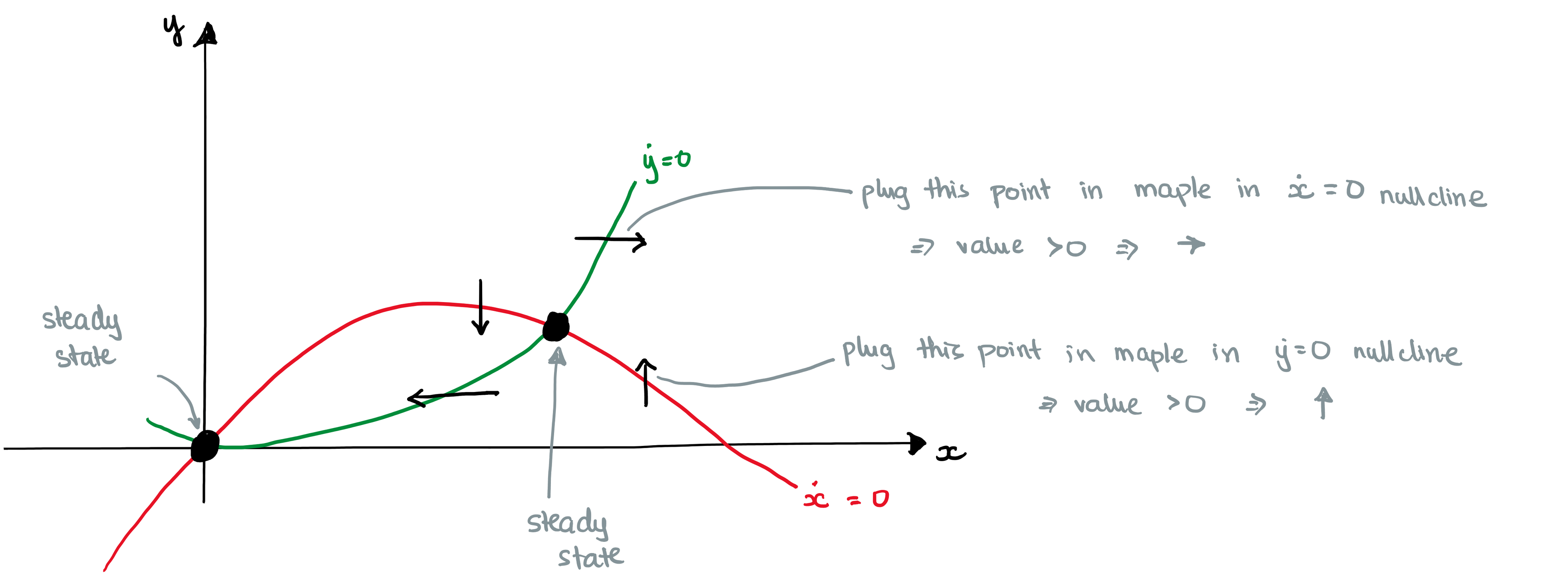
a) Find Nullclines

$\dot{x} = 0$ $\Leftrightarrow$ $0 = y[1 - y - p_a x] + x[y + x] - x[1 - (1-p_a)x]$

$\Leftrightarrow$ $0 = -y^2 + [1 - p_a x + x]y + [x^2 - x - (1-p_a)x^2]$ ← quadratic

$\dot{y} = 0$ $\Leftrightarrow$ $0 = x[y + p_a x] - y[1 - y - p_a x]$

$\Leftrightarrow$ $0 = y^2 + [x - 1 + p_a x]y + p_a x^2$ ← quadratic



b) Steady States

↳ points where $\dot{x} = \dot{y} = 0$ (or when graphs intersect)

c) Flow Field

- on $\dot{x} = 0$ nullcline, only y changes ⇒ flow lines are ↑ or ↓ (+) or (−)
- " $\dot{y} = 0$ " " x " " → or ←

Exercises

- Find nullclines , steady states & sketch phase plane diagram

a) $\dot{x} = x^2 - y$, $\dot{y} = 2xy - 1$

b) $\dot{x} = x^2 - xy$, $\dot{y} = x + y - 1$

c) $\dot{x} = x\sin(y)$, $\dot{y} = x^3 - y$

a) Nullclines

$\dot{x} = 0 = x^2 - y$ $\Leftrightarrow$ $y = x^2$

$\dot{y} = 0 = 2xy - 1$ $\Leftrightarrow$ $y = \dfrac{1}{2x}$