Achievable Rates of Multi-Carrier Modulation Schemes for Bandlimited IM/DD Systems

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Abstract—In this paper, we comprehensively investigate the achievable rates of selected band-limited intensity modulation schemes, which are important for optical wireless communication (OWC) applications, while accounting for the specific nature of their signal construction (non-negative, real and baseband), and imposing identical bandwidth and average optical power constraints. Furthermore, we identify/design methods to effectively trade between these parameters. Three variants of orthogonal frequency division multiplexing (OFDM), namely Asymmetrically Clipped Optical OFDM (ACO-OFDM), spectrally and energy efficient OFDM (SEE-OFDM), and DC biased optical OFDM (DCO-OFDM) are studied. The clipping noise in ACO-OFDM and SEE-OFDM is found to consume a large excess bandwidth. The detrimental effects of this excess bandwidth on the achievable rate are evaluated. For SEE-OFDM, the problem of optimal power allocation among its components is formulated and solved using the Karush-Kuhn-Tucker (KKT) method. For DCO-OFDM, the clipping noise is modeled and incorporated in the analysis. Among the existing schemes, DCO-OFDM yields the best overall performance, due to its compact spectrum. In order to improve the achievable rate, we propose and analyze two improved distortionless variants, filtered ACO-OFDM (FACO-OFDM) and single-carrier pulse amplitude modulation (SC-PAM) are studied. The clipping noise is modeled and incorporated in the analysis. Among the existing schemes, DCO-OFDM yields the best overall performance, due to its compact spectrum. In order to improve the achievable rate, we propose and analyze two improved distortionless variants, filtered ACO-OFDM (FACO-OFDM) and filtered SEE-OFDM (FSEE-OFDM), which yield better spectral efficiency than ACO-OFDM and SEE-OFDM, respectively. FSEE-OFDM, being the most spectrally efficient, outperforms all schemes.

I. INTRODUCTION

Optical wireless communication (OWC) employing white light-emitting diodes (LEDs) has gained great attention for indoor applications, in what is nowadays known as visible light communication (VLC) [2], [3]. It is rapidly evolving as a viable technology for simultaneous illumination and communication, thus exploiting the license-free optical spectrum for communication without interfering with existing radio frequency (RF) systems. The main focus has been towards intensity modulation and direct detection (IM/DD) due to its low-cost and simplicity.

Unlike RF systems, IM/DD systems only allow positive, real-valued baseband signals constrained by an average optical power mainly to fulfill a dimming requirement. Thus, the construction of such signals differs from conventional RF signals. For instance, in orthogonal frequency division multiplexing (OFDM) schemes, Hermitian symmetry in frequency domain and an appropriate operation in time domain are required to ensure a real non-negative signal, which are not necessary in RF.

Several optical OFDM schemes have been proposed in the literature. One example is the asymmetrically clipped optical OFDM (ACO-OFDM) scheme, which activates only odd sub-carriers to carry the complex valued symbols [4], followed by clipping of time-domain signal at zero level to eliminate the negative part of the signal without inducing any interference on the odd sub-carriers. The information rate of ACO-OFDM under an average optical power constraint was studied in [5]. Another scheme is the DC biased optical OFDM (DCO-OFDM) [6], where a unipolar signal is generated by clipping and DC biasing. Comparative studies of ACO-OFDM and DCO-OFDM have shown that DCO-OFDM is less efficient in terms of power [6]. Other optical OFDM schemes with different spectral or energy properties include the pulse-amplitude modulation (PAM) discrete multi-tone (PAM-DMT) scheme [7] which modulates all sub-carriers with imaginary-valued symbols and applies a clipping operation, the Flip-OFDM scheme [8] which modulates all sub-carriers with complex data symbols and sends the non-negative and inverted negative parts of the signal over two transmissions, and the spectrally and energy efficient OFDM (SEE-OFDM) scheme [9] which uses all sub-carriers by superimposing multiple ACO-OFDM-like components. We note that most previous works on these schemes do not take their effective bandwidth requirement into account; rather the focus has been mainly on optical power efficiency.
the best of our knowledge, the effects of clipping noise on required bandwidth has not been taken into account while evaluating their rates. OFDM techniques have been studied for discrete-time channels. However, due to clipping in ACO and SEE-OFDM, their bandwidth requirement increases and their spectral efficiency drops. In this work, we are interested in investigating the effects of this signal construction on the achievable rate. Further, we are interested in finding a way to remedy this spectral efficiency loss incurred due to clipping, and to retain the promised performance of these OFDM techniques.

Recently, a comparison of these unipolar OFDM schemes is presented in [10], in which asymptotic lower bounds on achievable rate for independent Gaussian inputs and unequal power allocation for SEE-OFDM with multiple components is evaluated. This involves an empirically chosen value of optical power allocation for each component which is theoretically proven to lower bound the information rate. However, this is an asymptotic result using all possible components, whereas, in this work (Section IV), using the KKT method, we analytically prove that this unequal power allocation depends on the SNR and all components are not necessarily required for maximum achievable rate at a given SNR. Rather, we show that at lower SNR, we can achieve a higher achievable rate by using fewer components (see Fig. 13 and Table I).

Other works on the achievable rates for various optical OFDM schemes can be found in [11]–[14]. In [11], a distortion measure, namely error vector magnitude (EVM), is used to characterize distortion in ACO-OFDM and DCO-OFDM, which is then used to find an optimal clipping ratio. These parameters are used with Bussgang theorem to find the achievable rate. However, the nature of clipping in ACO-OFDM and DCO-OFDM are quite different and the resulting information rates do not reflect the performance floor in DCO-OFDM due to clipping noise. In [12], information rates of ACO-OFDM and DCO-OFDM are analyzed under an electrical power constraint and in the presence of LED’s non-linear distortion. Methods for pre-distortion and linearization to remedy the non-linear distortion introduced by optical front-end are proposed and achievable rates for the system employing these methods are analyzed. Since the electrical power constraint is used, the results do not reflect the effect of DC bias, and thus the average optical power, on achievable rate, which is especially important in DCO-OFDM. In [13], lower bounds on the channel capacity of asymmetrically and symmetrically clipped optical (ASCO)-OFDM and asymmetrically clipped DC biased optical (ADO)-OFDM are analyzed by evaluating mutual information using the PDFs of time domain ADO- and ASCO-OFDM signals. However, this approach is not informative in terms of symbol encoding, because the PDF of time-domain optical OFDM variants is always truncated Gaussian distribution regardless of symbol encoding in frequency domain. Moreover, the time-domain signal samples are correlated in an OFDM signal, due to which the mutual information method may not be readily applicable. This method also does not incorporate the effects of clipping distortion. In [14], a preliminary achievable rate analysis for layered ACO-OFDM (LACO-OFDM), which is similar to SEE-OFDM, is presented and it is shown via simulations that the number of layers in LACO-OFDM (or components in SEE-OFDM) should adaptively change with SNR. However, the optimization of power allocation among the layers at a given SNR is not analyzed. The excess bandwidth issue in optical OFDM variants is not dealt with in any of these works.

As an alternative to optical OFDM schemes, one can use single-carrier schemes with appropriate pulse-shaping. These schemes include pulse position modulation (PPM) [15], on-off keying (OOK) [16] and PAM [17], [18]. Such single carrier schemes have relatively lower design complexity compared with optical OFDM schemes. The question which arises here is: Do single carrier schemes also have an advantage in terms of spectral efficiency when bandwidth is taken into account? In most previous works, time-disjoint pulses are considered due to the simplicity of implementation [19], [20]. Previous works on PAM consider using rectangular pulses [17], [18], [20] which requires a large bandwidth. In [21], Nyquist pulses are considered that are shown to provide benefits in achievable rates. In this work, we will consider time-disjoint pulses with superior time-frequency concentration properties, which provide a good trade-off between complexity and achievable rate.

In this paper, we present a comparative study of the effective information-rates of ACO-OFDM, SEE-OFDM and DCO-OFDM under a bandwidth constraint and an average optical power constraint. Moreover, we study single-carrier PAM (SC-PAM) using pulse shapes with good time-frequency concentration properties, so that the information rate in a more realistic scenario can be found. The ACO-OFDM, SEE-OFDM and DCO-OFDM signals are also analyzed in terms of their transmission bandwidth, in order to better reflect their spectral efficiencies under a bandwidth constraint. Of particular interest is the clipping operation that leads to an overflow of frequency components beyond the bandwidth of the bipolar OFDM signal. To ensure that the clipped signal is contained in the available bandwidth, one has to maintain a back-off between the bandwidth of the bipolar OFDM signal and the available LED’s bandwidth, which further reduces the spectral efficiency. It turns out

1We note that other schemes can be analyzed using the same framework.
that this overflow can double the bandwidth in ACO-OFDM and SEE-OFDM, but has less impact in DCO-OFDM. However, DCO-OFDM suffers from a non-linear distortion introduced by clipping. These effects are incorporated in the information rate analysis to get a realistic estimate of achievable rate with these schemes. In SEE-OFDM, since there are many ACO-OFDM-like components, achievable rate at a given signal-to-noise ratio (SNR) is affected by the distribution of optical power among these components. To find this optimal power allocation, we use KKT method to maximize the achievable rate at a given optical SNR. We also compare the OFDM variants with SC-PAM.

In our preliminary work [1], we presented a comparative study of the achievable rates SC-PAM, ACO-OFDM and DCO-OFDM, which suggested that the transmission bandwidth is the main factor limiting the achievable rate. In this work, we extend the investigation to SEE-OFDM by evaluating and optimizing the achievable rate via optimal power allocation using the KKT method. Furthermore, in the initial work [1], it was observed that due to spectral compactness, DCO-OFDM has the best overall performance among the existing modulation schemes. Motivated by this observation and to remedy the bandwidth-wastefulness of ACO- and SEE-OFDM and to avoid the distortion in DCO-OFDM, we present two improved spectrally efficient, distortionless variants, namely filtered ACO-OFDM (FACO-OFDM) and filtered SEE-OFDM (FSEE-OFDM). These schemes improve the construction of ACO- and SEE-OFDM by removing the out-of-band clipping noise from ACO/SEE-OFDM and adding a DC bias to ensure non-negativity. A comprehensive study of these improved schemes is conducted to study the effect of low-pass filtering and DC bias. FSEE-OFDM, being the most spectrally efficient and distortionless, is found to outperform all the existing schemes in terms of achievable rate.

The rest of this article is organized as follows: In Section II, IM/DD system model is discussed and power and bandwidth constraints are defined. In Section III, analysis for ACO-OFDM is presented, along with the design and analysis of FACO-OFDM. In Section IV, achievable rate of SEE-OFDM is formulated and maximized via optical power allocation. Design of FSEE-OFDM is also presented. In Section V, achievable rate of DCO-OFDM is found. In Section VI, the achievable rate of SC-PAM is analyzed. In Section VII, the achievable rates of all the existing and proposed schemes are compared and the results are discussed. Finally, Section VIII draws conclusion from this work.

II. IM/DD SYSTEM MODEL AND PROBLEM STATEMENT

A. Optical Intensity Channel

Fig. 1 shows the block diagram of an IM/DD system, where the information is transmitted by modulating the intensity of an LED. At the receiver, inexpensive optical intensity detectors are used.

Let \( x(t) \geq 0 \) be the electrical current applied to LED’s input, which is linearly transformed into an optical intensity signal \( s(t) = G x(t) \), where \( G \) (Watts/Ampere) is the electrical-to-optical conversion factor. We assume that the LED is operating in its linear region of operation. We assume that the OWC channel \( h(t) \) is a frequency-flat channel. Thus, the photodetector at the receiver receives \( s(t) \) and converts it to an electrical signal \( r(t) = Rs(t) \), where \( R \) (Amperes/Watts) is the responsivity of the photo-detector. We assume that \( R = G = 1 \) without loss of generality, and hence \( \int s(t) = s(t) = x(t) \). This signal is corrupted by noise \( w(t) \) in the electrical domain, so that the received electrical signal can be written as

\[
y(t) = x(t) + w(t).
\]

Here \( w(t) \) is AWGN with zero mean and variance \( \sigma^2_w \), due to the fact that the electronic noise and the high intensity shot noise are the dominant noise sources and are both additive, white in the band of interest, and approximately Gaussian distributed by a central limit theorem (CLT) argument [6].

In case of a frequency selective channel, the water-filling algorithm can be used to find the achievable rate. In order to do this, the average optical power will need to be converted into the corresponding average electrical power of the data-bearing subcarriers. This average electrical power and the frequency response of the channel can be used to allocate the electrical power to individual data-bearing subcarriers using the water-filling algorithm to find the achievable rate.

B. Constraints

Due to the IM/DD operation, \( s(t) \) and hence \( x(t) \) must be nonnegative real, constrained by an average optical power in view of the lighting function and safety considerations. This average optical constraint can be written as

\[
E \left[ \frac{1}{T_s} \int_{nT_s}^{(n+1)T_s} x_n(t) dt \right] = P_o.
\]
where \( T_s \) is the symbol duration and the expectation is with respect to \( n = 1, 2, \ldots, \) i.e., the integrals for all possible realizations of \( x_n(t) \) are averaged. We define the SNR of the system as \( 10 \log (P_o/\sigma_w) \) dB, following the previous work [20]. Note that the received electrical signal \( y(t) \) can be negative valued due to \( u(t) \).

The bandwidth of the system is also limited due to the low-pass characteristics of LEDs [22]. For a fair comparison under practical conditions, we define a fractional energy bandwidth (FEB) \( B_e \) that contains \((1-\epsilon)\) times the total energy of the signal\(^2\), i.e., \( B_e \) satisfies the following condition:

\[
\frac{1}{B_e} \int_{-B_e}^{B_e} |X(f)|^2 df \left[ \int_{-\infty}^{\infty} |X(f)|^2 df \right]^{-1} = 1 - \epsilon \tag{3}
\]

where \( X(f) \) is the continuous time Fourier transform (CTFT) of the signal \( x(t) \). This \( B_e \) will be estimated from the discrete Fourier transform (DFT) of an oversampled signal. The FEB should satisfy a bandwidth requirement given by \( B_e < B \) where \( B \) is the bandwidth of the LED. If for some transmit signals \( B_e > B \), then an outage is declared, and is taken into account in rate computation. In the next sections, ACO-OFDM, SEE-OFDM, DCO-OFDM and SC-PAM are investigated in depth for the effects of signal construction on their respective achievable rates.

III. ASYMMETRICALLY CLIPPED OPTICAL OFDM

ACO-OFDM

In this section, ACO-OFDM system is first described, followed by the modeling of clipping distortion, which further leads to the evaluation of achievable rate. Motivated by the insights from this study, Filtered ACO-OFDM (FOFO-OFDM) is proposed and its achievable rate is studied in the presence of identical power and bandwidth constraints.

A. Achievable Rate of ACO-OFDM

1) Odd Sub-carrier Modulation: Fig. 2 shows the block diagram of an ACO-OFDM system. Constellation symbols are assigned to \( X_k \), where \( k \) is odd and \( 0 \leq k \leq N/2 - 1 \), while even sub-carriers are set to zero. The complex symbol vector \( \mathbb{X} = [0, X_1, 0, X_3, 0, \ldots, X_{N-1}] \) is Hermitian symmetric to ensure a real signal, i.e., \( X_{N-k} = X_k^* \), \( k = 1, 3, \ldots, \frac{N}{2} - 1 \). In order to generate an oversampled signal with an oversampling factor \( L \), \( N(L-1) \) zeros are inserted in the middle of this vector to get the IDFT input \( \mathbb{X} = [0, X_1, 0, X_3, 0, \ldots, X_{N/2-1}, 0, 0, 0, \ldots, X_{N-1}] \). The reason for oversampling is explained in the following section. The time domain signal is generated by applying the inverse discrete Fourier transform (IDFT) on \( \mathbb{X} \) leading to [23], [24]

\[
x[n] = \frac{1}{\sqrt{NL}} \sum_{k=1}^{N-1} X_k \exp \left( \frac{j2\pi kn}{NL} \right) \tag{4}
\]

where \( n \in \{0, \ldots, NL - 1\} \). Due to odd sub-carrier modulation, \( x_n \) has odd-symmetry, i.e., \( x_{n+N/2} = -x_n \).

2) Asymmetric Clipping: In order to get a unipolar signal, \( x_{BP}[n] \) is clipped at zero level to get \( x_{ACO}[n] \) as follows:

\[
x_{ACO}[n] = (x_{BP}[n])^+ = 0.5 \times (x_{BP}[n] + |x_{BP}[n]|) \tag{5}
\]

where \((x)^+ = \max\{0, x\}\). Since \( x_{BP}[n] \) has odd symmetry, \(|x_{BP}[n]| \) has even symmetry. Hence, \(|x_{BP}[n]|\) has energy at even sub-carriers only, which means it is orthogonal to data carriers [4]. Therefore, this clipping operation does not distort the data-bearing subcarriers. This signal \( x_{ACO}[n] \) is then passed through the DAC to get the analog ACO-OFDM signal \( x_{ACO}(t) \).

It should be noted here that if the clipping is done in discrete-time domain, the D/A converter will result in negative signal regrowth due to interpolation and the resulting analog signal will not be transmittable by the LED. However, if the discrete-time signal is sufficiently oversampled, it closely resembles the analog signal. If this signal is clipped and passed through DAC, the negative signal regrowth is negligible. In OFDM literature, it has been shown that the envelope characteristics of an OFDM signal converge for an oversampling factor \( L \geq 4 \), and therefore \( L = 4 \) is used as a convention [23]–[25]. Another option is to clip the signal after D/A [26]. However, in this case, if the circuit used to perform the clipping, including the LED, does not have a sufficiently large bandwidth (we later see in Fig. 3, Section III, that the LED’s bandwidth should be almost twice of the band occupied by data-bearing subcarriers to carry the non-negative signal with negligible distortion), the signal will undergo distortion. In this work, in order to quantify the bandwidth and devise methods to use it more efficiently, we use the convention of clipping an oversampled signal in discrete-time. Since it is stipulated that the negative regrowth in this case will be negligible for \( L = 4 \) [23]–[25], we assume that the corresponding analog signal will have same envelope properties as the oversampled

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\(^2\)FEB is defined in RF systems in order to minimize spectral-leakage to neighboring bands. Contrary to RF, this is defined here in order to reduce the distortion incurred on the signal by the light source due to its limited bandwidth.
signal. Fig. 2 shows the system model of ACO-OFDM model using this convention. Oversampling has been done through zero insertion in frequency domain [23]. It should be noted here that clipping after DAC, as in [26], will have similar results in basic ACO-OFDM system if the signal before clipping occupies only half of the LED’s bandwidth.

3) Probability Density Function (PDF) of ACO-OFDM Signal: The signal \( x_{BP}(t) \) is generated by the addition of a large number of subcarriers. By the central-limit theorem (CLT) approximation, this signal is zero mean Gaussian with variance equal to the average power allocated to the sub-carrers. After clipping, all the negative amplitudes are set to zero, which changes the signal distribution to a truncated Gaussian distribution.

\[
f_{x_{ACO}}(x) = 0.5\delta(x) + \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{x^2}{2\sigma_x^2}\right) u(x),
\]

where \( \sigma_x^2 \) is the average power of the signal before clipping, \( \delta(x) \) is the Dirac delta function and \( u(x) \) is the unit step function. The factor 0.5 follows due to the odd-symmetry of \( x_{BP}(t) \). Parseval’s theorem for the unitary form of IDFT in (4) ensures

\[
\mathbb{E} \left[ \sum_{n=0}^{N-1} |x_n|^2 \right] = \mathbb{E} \left[ \sum_{k=0}^{N-1} |X_k|^2 \right] = N\sigma_x^2.
\]

Let \( P_e \) be the average electrical power allocated to the data-bearing sub-carriers before clipping. Note that before clipping, \( \mathbb{E}[|X_k|^2] = P_e \) for odd \( k \) and \( \mathbb{E}[|X_k|^2] = 0 \) for even \( k \), then

\[
\sigma_x^2 = \frac{1}{N} \sum_{k=0}^{N-1} \mathbb{E}[|X_k|^2] = \frac{P_e}{2}.
\]

The average optical power of the signal is found as follows:

\[
P_o = \int_{-\infty}^{\infty} f_{x_{ACO}}(x) dx = \frac{\sigma_x}{\sqrt{2\pi}}.
\]

Now, using (7) and (8), \( P_e \) and \( P_o \) are related as follows [5]:

\[
\sigma_x^2 = 2\pi P_o^2 \quad \text{and} \quad P_e = 4\pi P_o^2.
\]

This allows us to transform the optical power to an electrical power \( P_e \) that can be allocated across the subcarriers. Next, we study the bandwidth requirements of ACO-OFDM.

4) Effect of Clipping on Required Bandwidth: The clipping operation in ACO-OFDM introduces the component \( |x_{BP}[n]| \) in (5). For distortionless transmission of the unipolar signal, we need sufficient bandwidth to transmit both the signal components in (5). Fig. 3 shows DFTs of the two components. The signal is oversampled by a factor \( L = 4 \) prior to clipping to observe high-frequency contents of the analog signal. We see that the data-bearing sub-carriers remain uncorrupted due to the orthogonality of two signal components. However, \( |x_{BP}[n]| \) occupies a significantly larger bandwidth as compared to \( x_{BP}[n] \), which increases the total bandwidth requirement of the ACO-OFDM signal. If we filter out this spectral overflow, the signal will not be unipolar, as shown in Fig. 4, and hence not transmittable by the LED without distortion. The LED will introduce distortion in the signal as it will limit both its amplitude and frequency.

In order to understand the reason for this increased bandwidth requirement, consider the unipolar continuous-time ACO-OFDM signal in (5). The clipping operation in the ACO-OFDM signal can be interpreted as a multiplication by a rectangular pulse train. The component \( |x_{BP}(t)| \) can be expressed as a product of \( x_{BP}(t) \) with a train of non-overlapping rectangular pulses with varying duration and polarity, namely

\[
|x_{BP}(t)| = x_{BP}(t) \sum_{r=1}^{R} m_r \text{rect} \left( \frac{t - D_r}{T_r} \right),
\]

where \( D_r \pm T_r/2 \) mark the zero-crossings of \( x_{BP}(t) \), \( m_r = 1 \) if \( x_{BP}(t) \geq 0 \) for \( t \in [D_r - T_r/2, D_r + T_r/2] \) and \( m_r = -1 \) otherwise. The variables \( D_r, R, \) and \( T_r \) are all random variables depending on \( X_k \). The value of \( T_r \) in (10) can be as small as one sampling interval \( T_s/N \).
of the ACO-OFDM signal, that we observe for a given realization of the signal and use to analyze the systems effective data rate. Its definition depends on the systems tolerance against an approximation error resulting from neglecting the spectral leakage. In this work, we have approximated/chosen \( N/T_s \) Hz as the signal bandwidth and treated the signal energy outside this band as spectral leakage, which is a very small fraction of the signal energy and can be neglected, as in conventional OFDM.

6) Achievable Information Rate of ACO-OFDM: The information rate of ACO-OFDM subject to an average optical intensity constraint was analyzed in [5] by only considering the bandwidth occupied by the data-bearing sub-carriers. The excess bandwidth requirement introduced by the clipping operation was not incorporated in the analysis. Here, we aim to study the effect of this signal construction on the information rate.

In order to find the information rate, the optical intensity constraint is converted to an electrical energy constraint using (9). Information rate per sub-carrier \( C_{ACO}^{(sc)} \), with complex Gaussian input symbol, is found as follows [5]:

\[
C_{ACO}^{(sc)} = \log_2 \left( 1 + \frac{P_e}{4\sigma_w^2} \right) \text{bits/subcarrier.}
\]

Note that the power allocated to the odd sub-carriers \( P_e \) is reduced to \( P_e/4 \) due to the scaling of \( x_{BP}(t) \) by 0.5 in (5). Now, every OFDM symbol comprises of \( N/4 \) data-bearing sub-carriers, that are transmitted in \( T_s \) seconds. Hence, for every symbol transmitted in \( T_s \) seconds, rate per second is given as:

\[
C_{ACO}^{(sec)} = \frac{N}{4T_s} \log_2 \left( 1 + \frac{\pi P_o^2}{\sigma_w^2} \right) \text{bits/sec.} \tag{11}
\]

It can be seen from Figs. 3 and 6 that a bandwidth greater than \( N/2T_s \) Hz is occupied by an ACO-OFDM signal. For an available bandwidth of \( B \geq N/2T_s \) Hz, only those symbols can be transmitted for which \( B_s \leq B \). Therefore, the effective information rate for ACO-OFDM scheme is found as follows:

\[
C_{ACO} = \frac{N}{4T_s B} \log_2 \left( 1 + \frac{\pi P_o^2}{\sigma_w^2} \right) \Pr[B_s \leq B] \text{bits/sec/Hz,} \tag{12}
\]

where \( \Pr[B_s \leq B] \) is obtained from Fig. 6. We see that ACO-OFDM symbols may occupy nearly \( N/T_s \) Hz (for \( \epsilon = 0.005 \)), which is twice the bandwidth occupied by data-bearing sub-carriers. Therefore, in the numerical comparison, we will use \( B \approx N/T_s \) and \( \Pr[B_s \leq B] \approx 1 \) in (12).

B. Filtered ACO-OFDM

In this section, we build further upon our observations above to propose a novel variant of ACO-OFDM,
namely filtered ACO-OFDM (FACO-OFDM) to deal with the spectral overlap of ACO-OFDM, and study the corresponding achievable rate. In the previous section, the discussion on ACO-OFDM signal’s characteristics clearly shows that the signal’s bandwidth is not being used efficiently. Here, by designing and investigating FACO-OFDM, we aim to improve the bandwidth utilization while preserving the desirable characteristics of ACO-OFDM, i.e., distortionless transmission and simple receiver design.

We see in Fig. 4 that low-pass filtering leads to a bipolar and hence an in-transmittable signal. If this signal is clipped a second time to eliminate the negative regrowth, then the clipping noise will not be orthogonal to the signal. This is because the filtered signal does not have the property of odd-symmetry, which means that the clipping noise from second clipping will not fall on even sub-carriers only. As a result, the data symbols will become distorted. However, if we add a DC bias to the filtered signal, that completely removes the negative parts of the signal, then the signal will become transmittable without any distortion. The block-diagram for the proposed FACO-OFDM is shown in Fig. 7. The filtered and DC-biased FACO-OFDM symbol is expressed as follows:

$$x_{\text{FACO}}[n] = x_{\text{ACO}}^{(\text{LP})}[n] + \min\{x_{\text{ACO}}^{(\text{LP})}[n]\},$$

where $x_{\text{ACO}}^{(\text{LP})}[n]$ is the signal at the output of low-pass filter (LPF) with a cut-off frequency $B_{\text{LP}}$, i.e.,

$$X_{\text{ACO}}^{(\text{LP})}[k] = X_{\text{ACO}}[k] \times H_{\text{LP}}[k],$$

where the filter $H_{\text{LP}}[k]$ is essentially a rectangular window multiplied with the DFT of signal, i.e., $X_{\text{ACO}}[k]$.

$$H_{\text{LP}}[k] = \begin{cases} 1, & \text{for } k \in [0, B_{\text{LP}} T_s] \cup [NL - 1 - B_{\text{LP}} T_s, NL - 1] \quad \text{(15)} \\ 0, & \text{otherwise} \end{cases}$$

$\beta_{\text{DC}}$ in (13) is the DC bias added to the filtered signal to make it transmittable. The implementation will be discussed in Section III-B2. The studies in [24], [25] show that the minimum amplitude estimate will be realistic when an oversampling factor $L = 4$ is used.

The added DC bias in FACO-OFDM is evidently input-dependent. However, since the DC bias does not interfere with data-bearing sub-carriers, it is not required to be communicated to the receiver. Rather the odd sub-carriers can be decoded independent of the DC bias. Therefore, no side information is required and the receiver remains same as the ACO-OFDM receiver. Moreover, we also notice in Fig. 4 that the negative regrowth of the signal depends on the cut-off frequency of the low-pass filter. The larger the cut-off frequency, the smaller the negative regrowth and hence smaller will be the required DC bias. Note that since the low-pass filter does not affect the DC carrier of $x_{\text{ACO}}(t)$, the optical power of FACO-OFDM is given as follows:

$$P_o^{(\text{FACO})} = E[\beta_{\text{DC}}] + P_o^{(\text{ACO})}.$$ 

Therefore, FACO-OFDM involves trading between bandwidth $B_{\text{LP}}$ of the signal and the average optical power $P_o^{(\text{FACO})}$. In the next section, we investigate the competing effects of these two parameters.

1) Achievable Rate of FACO-OFDM: In order to derive the achievable rate of FACO-OFDM, we first consider the rate of ACO-OFDM, given in (11). Using (16), we get

$$C_{\text{FACO}} = \frac{N}{4T_s} \log_2 \left( 1 + \pi \left( \frac{P_o^{(\text{FACO})} - E[\beta_{\text{DC}}]}{\sigma_w^2} \right)^2 \right) \text{bits/sec.}$$

(17)

Every FACO-OFDM symbol requires a bandwidth $B_{\text{LP}} \geq N/2T_s$. Therefore, the achievable rate becomes:

$$C_{\text{FACO}} = \frac{N}{4T_s B_{\text{LP}}} \times \log_2 \left( 1 + \frac{\pi (P_o^{(\text{FACO})} - E[\beta_{\text{DC}}])^2}{\sigma_w^2} \right) \text{bits/sec/Hz.}$$

(18)

Hence, the achievable rate of FACO-OFDM depends on the variables $\beta_{\text{DC}}$ and $B_{\text{LP}}$.

Now, we need to find the optimal compromise between the bandwidth and optical power. Fig. 8 shows the variation of $E[\beta_{\text{DC}}]$ with $B_{\text{LP}}$ and the empirical probability distribution of $\beta_{\text{DC}}$ for given $B_{\text{LP}}$. The cut-off frequency $B_{\text{LP}}$ is varied between $N/2T_s$, the bandwidth containing the data-bearing sub-carriers, and $N/T_s$, the 99.5% fractional energy bandwidth of ACO-OFDM. Fig. 9 shows the achievable rates of FACO-OFDM for various values of $B_{\text{LP}}$. The results indicate that an increase in bandwidth has a greater detrimental effect on the achievable rate than an increase in optical power. The best rate is achieved with $B_{\text{LP}} = N/2T_s$. This may be explained by noting that the excess bandwidth in ACO-OFDM ($N/2T_s \leq B \leq N/T_s$) contains a significantly smaller portion of signal energy, as evident from Fig. 3. Therefore, although the excess bandwidth is as large as the useful bandwidth, it amounts to a small increase in
the optical power. We also observe that the advantage of smaller bandwidth requirement is more effective at high SNR. This may be explained by noting that at lower SNR, optical power has a more significant competing effect on the achievable rate, as a result of which filtering may not yield a significant advantage at low SNR.

2) Implementation Considerations: FACO-OFDM essentially trades implementation complexity for increased achievable rate. This is due to the additional $\beta_{DC}$ estimator and the low-pass filter required in the FACO-OFDM system. The filter can be implemented as frequency-domain windowing or an equivalent time domain discrete-time filter with properly appended cyclic prefix [27, Fig. 2]. Moreover, an oversampled ACO-OFDM signal is required to implement FACO-OFDM, which is another factor that increases the complexity. In view of the previous studies on the OFDM [25], an oversampling factor of four (4) is sufficient to estimate the $\beta_{DC}$ and implement the filtering.

IV. SPECTRALLY AND ENERGY-EFFICIENT OFDM (SEE-OFDM)

In this section, we evaluate and optimize the achievable rate of SEE-OFDM. Motivated by the insights thereof, we propose and evaluate a novel variant filtered SEE-OFDM (FSEE-OFDM) to remedy the excess bandwidth requirement.

A. SEE-OFDM System

SEE-OFDM is another variant of optical OFDM proposed in [9]. In this scheme, many ACO-OFDM-like
signals are superimposed in such a way that the even sub-carriers are also utilized to carry data. Fig. 10 shows the block diagram of a SEE-OFDM transmitter with two components. The first component is same as an ACO-OFDM signal, i.e., it has useful data at odd sub-carriers and clipping noise at even sub-carriers. The second component utilizes the even sub-carriers. This is done by using only even-odd carriers (\(k = 2, 6, 10, \ldots\)) for data, which results in clipping noise only at even (\(k = 0, 4, 8, \ldots\)) sub-carriers [9]. At the receiver, successive decoding is used to recover the complete signal [9]. Building upon the same principle, a SEE-OFDM signal with \(R \in \{1, \ldots, \log_2(N/2)\}\) components is generated by adding \(R\) unipolar signal components \(x_{\text{SEE},r}[n]\):

\[
x_{\text{SEE}}[n] = \sum_{r=1}^{R} x_{\text{SEE},r}[n].
\]

The \(r\)th component \(x_{\text{SEE},r}[n]\) is generated by the \(N\)-point IDFT of \(X_k, r\). The \(N\)-point frequency domain vector \(X_r\) with symbols \(X_k, r\) has useful data at the indices \(2^{r-1}(2k - 1)\) for \(k \in \{1, 2, \ldots, N/2^{r+1}\}\) and is Hermitian symmetric. Similar to the ACO-OFDM case, \(N(L - 1)\) zeros are inserted in the middle of \(X_r\) for oversampling. \(X_r\). The clipping noise of \(r\)th component falls on indices \(2^r k\) for \(0 \leq k \leq \{0, 1, \ldots, N/2^{r+1} - 1\}\). The oversampled bipolar sequences \(x_{\text{SEE},r}[n]\), for all \(r = 1, 2, \ldots, R\), are clipped at zero level, added and passed through D/A converter. The composite, oversampled discrete-time signal with \(R\) components is given as follows:

\[
x_{\text{SEE}}[n] = \left[ \sum_{r=1}^{R} \left( \frac{1}{\sqrt{NL}} \sum_{k=0}^{N-1} X_k, r \exp \left( \frac{2\pi kn}{NL} \right) \right) \right]^+,
\]

for \(n = 0, \ldots, NL - 1\). The construction of SEE-OFDM signal is such that clipping noise of \(r\)th component is orthogonal to data-bearing sub-carriers of components \(\{1, 2, \ldots, r\}\) signal, but it is not orthogonal to components \(\{r + 1, \ldots, R\}\). At the receiver, first component \((r = 1)\) is decoded first. Using channel coding, we can guarantee that the signal of the first layer is decodable with a vanishingly small error probability, as long as we transmit below its capacity. In this case, the clipping noise can be reconstructed with high certainty. After reconstruction, the clipping noise is computed and subtracted from the total signal. At the next step, second component \((r = 2)\) is decoded and its clipping noise is subtracted from the remaining signal. In this way, each component of the signal is decoded in a successive fashion. An efficient implementation of a SEE-OFDM receiver can be found in [9]. The achievable rate using this scheme is discussed next.

1) Information Rate of SEE-OFDM: Consider the SEE-OFDM signal given by (20). Let \(P_{o,r}\) be the optical power of \(r\)th component, i.e., \(P_{o,r} = E[x_{\text{SEE},r}[n]]\). The optical power of the SEE-OFDM signal is the sum of average power of individual components, i.e., \(P_o = \sum_{r=1}^{R} P_{o,r}\). Although each of the \(R\) components modulates data on orthogonal sub-carriers (i.e., \(x_{\text{SEE},1}[n]\) has data symbols on subcarriers indexed \(k = 1, 3, 5, \ldots\) whereas \(x_{\text{SEE},2}[n]\) has data symbols on subcarriers indexed \(k = 2, 6, 10, \ldots\)), but due to the successive decoding at the receiver, estimation errors in one component introduces noise in the successive components. In order to find the achievable rate, we need to consider how this estimation error behaves. For this, we consider a SEE-OFDM signal with \(R = 2\). Let \(X_1\) be the symbol vector for \(x_{\text{SEE},1}[n]\), so \(X_1[k]\) has symbols at \(k = 1, 3, \ldots, N/2 - 1\). After the clipping, the DFT of \(x_{\text{SEE},1}[n]\) can be written as follows:

\[
x_{\text{SEE},1}[k] = 0.5 \times X_1[k] + Z_1[k],
\]

where \(X_1[k]\) and \(Z_1[k]\) are orthogonal to each other, i.e., \(Z_1[k]\) has non-zero values only for \(k = 0, 2, 4, \ldots, N/2\). Similarly, the second component is written as follows:

\[
x_{\text{SEE},2}[k] = 0.5 \times X_2[k] + Z_2[k],
\]

where \(X_2[k]\) and \(Z_2[k]\) are orthogonal to each other, i.e., \(X_2[k]\) has non-zero values only for \(k = 2, 6, 10, \ldots\) and \(Z_2[k]\) has non-zero values for \(k = 0, 4, 8, \ldots\). However, \(Z_2[k]\) is not orthogonal to \(Z_1[k]\), and in turn, \(X_{\text{SEE},1}[k]\). In general, \(Z_{i+1}[k]\) is not orthogonal to \(X_{\text{SEE},i}[k]\). The composite signal with \(R = 2\) at the receiver, after DFT, is expressed as follows:

\[
R[k] = 0.5 \times X_1[k] + Z_1[k] + 0.5 \times X_2[k] + Z_2[k] + W[k],
\]

where \(W[k]\) is white Gaussian noise. The decoding at the receiver begins by detecting the subcarriers in the first component, i.e., \(X_1[k]\). In the presence of a perfect channel code, \(X_1[k]\) will be detected without errors, which means that the effect of noise \(W[k]\) will be negligible [10]. The vector \(X_1[k]\) will pass through IDFT, clipped at zero level, and then passed through DFT again to recover the component \(X_{\text{SEE},1}[k]\). It should be noted here that since we aim to find the achievable rate, we are assuming here that if we operate at a rate lower than the achievable rate, then \(X_1[k]\) can be coded with a perfect channel code that will reduce the error rate to an arbitrarily small value [29], which would ensure perfect recovery of \(X_1[k]\) and in turn \(X_{\text{SEE},1}[k]\). This component is then subtracted from the total signal \(R[k]\) to get the residual as follows:

\[
R'[k] = R[k] - X_{\text{SEE},1}[k] = X_2[k] + Z_2[k] + W[k]
\]
Since $X_2[k]$ and $Z_2[k]$ are orthogonal, and $X_2[k]$ is also orthogonal to any other components of SEE-OFDM, if any $[28]$, $X_2[k]$ can be recovered in this step. Again, if we are operating at a rate lower than the achievable rate, then $X_2[k]$ will be recovered perfectly. The same decoding procedure may be repeated successively in case of more than two components.

It is clear from the above discussion that if the operating rates of $X_r[k]$ are lower than their respective achievable rates, henceforth denoted by $C_{\text{SEE},r}$, then there will be no estimation errors and the interference among the SEE-OFDM components will be completely eliminated at the receiver. Therefore, we can treat the SEE-OFDM components as independent of one another for evaluating the total achievable rate. Hence, the achievable rate of the SEE-OFDM is the sum of information rates of individual components:

$$C_{\text{SEE}} = \sum_{r=1}^{R} C_{\text{SEE},r}. \quad (25)$$

Now, we consider the achievable rate of the $r^{th}$ component. The $r^{th}$ component of the SEE-OFDM signal has $N/2^{r+1}$ data-bearing sub-carriers, and an additional $N/2^{r+1}$ nonzero subcarriers due to hermitian symmetry, for a total of $N/2^r$ nonzero subcarriers in the $r^{th}$ component. Following the same reasoning as in Section III-A6 used for ACO-OFDM signal, we can approximate the optical power $P_{o,r}$ in terms of electrical power allocated to the sub-carriers as a simple extension of (9):

$$\sigma_{x,r}^2 = \frac{P_{e,r}}{2} \quad \text{and} \quad P_{o,r} = \frac{\sigma_{x,r}}{\sqrt{2\pi}} \implies P_{o,r}^2 = \frac{P_{e,r}}{2} \frac{1}{2\pi}. \quad (26)$$

Note that this relationship was derived by assuming a Gaussian distributed time-domain OFDM signal, which is a consequence of the CLT. Therefore, this relationship applies more accurately for components with larger number of active subcarriers. Simulation results show that average optical power of components with four or more active carriers follow this relationship with reasonable accuracy. Since the components with one or two active sub-carriers contribute a very small fraction of optical power, so $P_o^{(\text{SEE})} = \sum_{r=1}^{R} P_{o,r}$ can be considered as approximately correct.

The achievable rate per active sub-carrier belonging to $r^{th}$ component, with data symbols following a complex Gaussian distribution, is given as follows:

$$C_{\text{SEE},r}^{(\text{sc})} = \log_2 \left( 1 + \frac{P_{e,r}}{4\sigma_w^2} \right)$$

$$= \log_2 \left( 1 + 2^{-1} \frac{P_{o,r}^2}{\sigma_w^2} \right) \text{bits/subcarrier.} \quad (27)$$

The $r^{th}$ component contains $N/2^{r+1}$ active subcarriers. Therefore, we can write the contribution of each SEE-OFDM component as follows:

$$C_{\text{SEE},r}^{(\text{comp})} = \frac{N}{2^{r+1}} \log_2 \left( 1 + 2^{r-1} \frac{\pi P_{o,r}^2}{\sigma_w^2} \right) \text{bits/component.} \quad (28)$$

Since $R$ components are transmitted as one symbol via parallel channels, the achievable rate is the sum of rates of all $R$ components. Therefore, the achievable rate of a SEE-OFDM symbol with $R$ components is given as follows:

$$C_{\text{SEE}}^{(\text{sym})} = \frac{N}{2} \sum_{r=1}^{R} \frac{1}{2r} \log_2 \left( 1 + 2^{r-1} \frac{\pi P_{o,r}^2}{\sigma_w^2} \right) \text{bits/symbol.} \quad (29)$$

Since SEE-OFDM uses similar signal construction as ACO-OFDM, we need to incorporate excess bandwidth considerations in this case as well. Fig. 11 shows the average DFT magnitude of a SEE-OFDM symbol. It can be seen that there is significant excess bandwidth occupied by the SEE-OFDM signal. Fig. 12 shows the bandwidth-outage probabilities for various fractional energy bandwidth considerations. Note that these probabilities were found using $R = \log_2(N/2)$ in all cases. In comparison with ACO-OFDM, FEB for SEE-OFDM is smaller. This can be understood by considering the SEE-OFDM spectrum in Fig. 11 and ACO-OFDM spectrum in Fig. 3. We see that SEE-OFDM clearly has a greater percentage of the total energy within the bandwidth $N/2T_s$, i.e., the bandwidth occupied by data-bearing carriers. This can be explained by noting that SEE-OFDM seeks to employ all the subcarrier for transmitting.
data symbols. As a result, all subcarriers for \( k \leq N/2 \) have data symbols. In ACO-OFDM, clipping noise and data occupy separate subcarriers. On the other hand, in SEE-OFDM, even subcarriers for \( k \leq N/2 \) have data as well as clipping noise from multiple SEE-OFDM components, whereas for \( k > N/2 \), there is only noise. As a result, SEE-OFDM has a larger fraction of signal energy inside \( N/2T_s \) Hz bandwidth. Therefore, as \( R \) decreases from \( \log_2(N/2) \), the outage probability curves in Fig. 6 approach the corresponding curves in Fig. 12. Unlike the average DFT of ACO-OFDM in Fig. 3, the noise component in SEE-OFDM does not decrease monotonically with frequency; rather there is larger energy at the even carriers (\( k = 0, 2, 4, \ldots \)) than even-odd carriers (\( k = 4, 8, 12, \ldots \)). This behavior results in smooth lines in Fig. 6 and bumpy lines in Fig. 12. Using the same reasoning as we did in case of ACO-OFDM, we get:

\[
C_{\text{SEE}} = \frac{N}{2T_s B} \sum_{r=1}^{R} \frac{1}{2} \log_2 \left( 1 + 2^{-2r-1} \pi P_{o,r}^2 \sigma_w^2 \right) \times \Pr[B_r \leq B] \text{ bits/sec/Hz. (30)}
\]

It can be seen that the achievable rate depends on the allocation of average optical power among the SEE-OFDM components. In order to get the maximum achievable rate from SEE-OFDM, we need to find an optimal power allocation. In the next section, we formulate and solve this optimization problem to obtain this optimal power allocation among the SEE-OFDM components at a given optical SNR.

2) Optimal Power Allocation: In this section, we aim to find a vector \( \mathbf{P}_o = [P_{o,1}, P_{o,2}, \ldots, P_{o,R}] \) that maximizes \( C_{\text{SEE}} \) given by (29). Therefore, we formulate the following optimization problem:

\[
C_{\text{SEE}}^{(\text{comp})} = \max_{\mathbf{P}_o} \left[ \frac{N}{2T_s B} \sum_{r=1}^{R} \frac{1}{2} \log_2 \left( 1 + 2^{-2r-1} \pi P_{o,r}^2 \sigma_w^2 \right) \right] \times \Pr[B_r \leq B] \text{ bits/sec/Hz, subject to}
\]

\[
\sum_{r=1}^{R} P_{o,r} = P_o \text{ and } P_{o,r} \geq 0 \text{ for } 1 \leq r \leq R. \quad \text{(31)}
\]

The function \( C_{\text{SEE}}^{(\text{comp})} \), given in (29), is not concave in \( P_{o,r} \) and hence the sum in (31) is also non-concave. Due to this non-concavity, this optimization problem is different from the standard water-filling power-allocation problem used to optimally allocate electrical power among OFDM sub-carriers [29] in the presence of multipath fading. For a concave function, since we have both equality and inequality constraints, we could use Karush-Kuhn-Tucker (KKT) conditions to obtain a unique solution. However, the non-concave nature of the objective function is prohibitive of a unique KKT-based solution. Nevertheless, we still use KKT conditions as they simplify the solution of the problem by finding a set of candidate solutions. The achievable rate in (31) will then be evaluated for all candidate solutions and the one yielding the maximum \( C_{\text{SEE}} \) will be selected.

The KKT Method: In order to apply the KKT method, we first write the Lagrangian associated with this optimization problem as follows:

\[
\mathcal{L} = -\sum_{r=1}^{R} \frac{1}{2\pi} \log_2 \left( 1 + 2^{-2r-1} \pi \frac{P_{o,r}^2 \sigma_w^2}{\sigma_w^2} \right) - \sum_{r=1}^{R} C_{r} - \sum_{r=1}^{R} \gamma_C \sum_{r=1}^{R} P_{o,r}
\]

\[
+ v \left( \sum_{r=1}^{R} P_{o,r} - P_o \right). \quad \text{(32)}
\]

Now, the KKT conditions associated with the Lagrangian are given as follows:

\[
\frac{\partial \mathcal{L}}{\partial P_{o,r}} = 0, \quad P_{o,r} \geq 0, \quad u_r \geq 0, \quad u_r P_{o,r} = 0 \quad \text{for } 1 \leq r \leq R \quad \text{(33)}
\]

In the next step, we find \( u_r \) by using the condition \( \frac{\partial \mathcal{L}}{\partial u_r} = 0 \):

\[
u_r = \frac{\pi P_{o,r}}{\sigma_w^2 + 2^{-2r-1} P_{o,r}^2} - v. \quad \text{(34)}
\]

Now we use \( u_r \) from (34) in condition \( u_r P_{o,r} = 0 \) to get the following:

\[
\frac{\pi P_{o,r}}{\sigma_w^2 + 2^{-2r-1} P_{o,r}^2} - v = 0, \quad \text{(35)}
\]

which is a cubic equation in \( P_{o,r} \), with following solutions:

- \( P_{o,r} = 0 \), or
- \( \frac{\pi P_{o,r}}{\sigma_w^2 + 2^{-2r-1} P_{o,r}^2} - v = 0 \), which is quadratic in \( P_{o,r} \),

and gives the following two possible solutions:

\[
P_{o,r}(v) = \frac{\pi \pm \sqrt{\pi^2 - 2^{-2r-1} v^2 \pi^2}}{2 \sqrt{v}}. \quad \text{(36)}
\]

In order to ensure that \( P_{o,r} \) is real, \( v \) must satisfy the following constraint:

\[
v \leq \frac{\sqrt{\pi}}{2 \sqrt{1} \sigma_w}. \quad \text{(37)}
\]

Thus, using the KKT conditions, we obtain the following candidate solutions:

\[
P_{o,r} \in \begin{cases} [0, \pm P_{o,r}(v)], & v \leq \frac{\sqrt{\pi}}{2 \sqrt{1} \sigma_w} \\ 0, & \text{otherwise}. \end{cases} \quad \text{(38)}
\]

The candidate solutions \( \mathbf{P}_o = [P_{o,1}, P_{o,2}, \ldots, P_{o,R}] \) are found by searching for solutions that satisfy \( \sum_{r=1}^{R} P_{o,r} = P_o \). The optimal solution is the one that
maximizes the information rate. Since each $P_{o,r}$, for $1 \leq r \leq R$, can have three possible values according to (38), we have $3^R$ candidate solutions. For all these candidates, we find $v \in [0, \frac{\sqrt{3}}{22+1+\pi w}]$ for which $\sum_{r=1}^{R} P_{o,r} = P_{o}$. We include this value of $v$ in our set of candidates, from which the solution that maximizes the objective function is selected.

3) Discussion on Results: Fig. 13 shows the achievable rate for SEE-OFDM with $R = 1, 2, 3, 4$ against the optical SNR. We observe that at lower optical SNR (SNR< 3 dB), the achievable rate is maximum if only the first SEE-OFDM component is turned on, while all the even carriers and hence $x_{\text{SEE}(t)}$ for $2 \leq r \leq R$, are set to zero. Thus at low SNR, the achievable rate of SEE-OFDM is the same as that of ACO-OFDM. As the SNR increases, we can successively turn on the next SEE-OFDM components. Table I shows the optimal results of $P_{o}$ for $R = 4$. These analysis results were validated by comparing them with the numerical solutions of the same optimization problem. If we define $\text{SNR}_{r,h,r}$ as the SNR at which $r^{th}$ component is turned on ($r \geq 2$), then we observe from the results that for SNR values close to $\text{SNR}_{r,h,r}$, $r^{th}$ component has smaller power per sub-carrier than $(r-1)^{th}$ component. However, as the SNR increases past $\text{SNR}_{r,h,r}$, all active sub-carriers have the same power.

In case of frequency selective channel, the allocated power values $P_{o,1}, P_{o,2}, \ldots, P_{o,R}$ will be further divided among the constituent subcarriers in every SEE-OFDM component individually using the water-filling algorithm. For example, $P_{o,1}$ will be used to calculate the corresponding electrical energy $\sigma_{e,1}^2$ using (26), and then this energy will be divided among the data-bearing subcarriers, i.e., the subcarriers with indices $k = 1, 3, 5, \ldots N/2 - 1$, using the water-filling algorithm [29].

Next, similar to FACOFD, we propose filtered-SEE-OFDM to cope with the spectral overflow of SEE-OFDM.

### B. Filtered SEE-OFDM

Since SEE-OFDM exhibits similar behavior in requiring excess bandwidth as ACO-OFDM, we apply the same type of filtering and DC biasing as discussed in Section III-B to get FSEE-OFDM signal:

$$x_{\text{FSEE}}[n] = x_{\text{SEE}}[n] + \min \{x_{\text{SEE}}[n] \}_{\beta_{\text{DC}}}$$

where the filtering operation is same as described in (14) and (15).

1) Achievable Rate of FSEE-OFDM: The achievable rate of FSEE-OFDM is found by incorporating following considerations in the analysis of SEE-OFDM:

- Setting the bandwidth per symbol to be $B = B_{LP}$ in (30).
- Using $P_{o}^{(\text{FSEE})} - E[\beta_{\text{DC}}] = \sum_{r=1}^{R} P_{o,r}$ in (30) and (31).

Hence the achievable rate of FSEE-OFDM is written as follows:

$$C_{\text{FSEE}} = \max_{x_{o}} \frac{N}{2T_{s}B_{LP}} \sum_{r=1}^{R} \log_{2} \left( 1 + \frac{2^{-1} \pi P_{o,r}^{2}}{\sigma_{e}^{2}} \right) \text{bits/sec/Hz}$$

where the maximization is with respect to $P_{o}$ such that $P_{o}^{(\text{FSEE})} - E[\beta_{\text{DC}}] = \sum_{r=1}^{R} P_{o,r}$.

Fig. 14a shows the average DC bias required for various number of components $R$ used in the FSEE-OFDM construction. We observe that with increasing $R$, the required DC bias is reduced. This can be explained as follows: The negative regrowth occurs due to interpolation around zero-valued samples. In ACO-OFDM ($R = 1$), the signal is zero valued 50% of the time. However, when many non-negative ACO-OFDM-like components are added together, zero-valued samples in the composite signal occur only when all the components are zero valued. If the components were independent and wide-sense stationary processes, the probability of zero-valued components would be $0.5^R$. However, conditions
of stationarity do not hold due to the nature of signal construction. Simulation results show that for \( R = 4 \), only 7.3% of the samples are zero valued. This behavior can be observed by comparing Figs. 4 and 15. This can also be understood by comparing the spectra of ACO-OFDM (Fig. 3) and SEE-OFDM (Fig. 11) and their FEBs (Figs. 6 and 12): since SEE-OFDM has a greater percentage of in-band power, removing the out-of-band components have a less significant effect on its non-negativity as compared to ACO-OFDM. Consequently, it exhibits smaller amount of negative signal regrowth after low-pass filtering, and hence requires a significantly smaller DC bias.

Fig. 14b shows the achievable rate for cut-off frequencies \( N/2T_s \leq B_{LP} \leq N/T_s \). We see that for \( B_{LP} = N/T_s \), \( C_{FSEE} \approx C_{SEE} \). The small difference is because there is no added DC bias in SEE-OFDM. We also observe that \( C_{FSEE} = C_{FACO} \) for \( SNR \leq 3\, dB \) and \( C_{FSEE} > C_{FACO} \) for \( SNR > 3\, dB \). The results clearly demonstrate that FSEE-OFDM significantly outperforms both SEE-OFDM and FACO-OFDM. The results for \( N = 256 \) are very close to those for \( N = 64 \) for any \( R \). The small difference can be attributed to the larger DC bias required for \( N = 256 \). Note that with \( N = 256 \), \( R \) can be up to 7. However, as we can predict from the trend in Fig. 13, the benefit of increasing \( R \) beyond 3 or 4 may not be significant (except at very high SNR), considering the corresponding increase in complexity at both the transmitter and the receiver [9].

2) Implementation Considerations: Like FACO-OFDM, FSEE-OFDM also requires a \( \beta_{DC} \) estimator and a LPF at the transmitter. It is important to note here that unlike ACO-OFDM, it is essential to keep the sampling rate at the receiver the same as that of the transmitter’s LPF. Otherwise, the noise components on even subcarriers (required for decoding) at the transmitter and the receiver will not be the same. This can be explained with the help of Fig. 16. For any sampling rate, the average clipping noise power is the same and the noise bandwidth is theoretically infinite. However, in discrete-time domain with a higher oversampling factor \( L \), this noise is spread over larger number of subcarriers, as shown in Figs. 3 and 11. For successive decoding, we need to recover the noise at the even subcarrier with \( k < N/2 \) and subtract from the total signal. Since SEE-OFDM theoretically occupies infinite bandwidth, the clipping noise that we observe in discrete time is the sum of low-frequency noise and aliased noise. In FSEE-OFDM, we have eliminated the clipping noise at frequencies higher that \( B_{LP} \). However, the first signal component \( x_{SEE,1[n]} \), that is recovered from the odd sub-carriers and clipping operation, will have clipping noise components at frequencies higher that \( B_{LP} \). In this situation, if the receiver’s sampling rate is smaller than the transmitter’s, i.e., \( L_R < L_T \), then the aliased noise in the total signal and in this recovered signal will not be

![Fig. 14](image_url)  
(a) Required DC bias for various values of \( B_{LP} \) and \( N \). (b) Achievable rate of FSEE-OFDM with \( R = 4 \) for various values of \( B_{LP} \).

![Fig. 15](image_url)  
Negative regrowth of SEE-OFDM signal due to low-pass filtering using a cut-off frequency \( N/2T_s \).
the same, which means the next component at even-odd sub-carriers will not be recovered properly.

V. DC BIASED OPTICAL OFDM (DCO-OFDM)

In this section, the signal processing and information-theoretic properties of DCO-OFDM are investigated. The clipping distortion is modeled and incorporated in the analysis.

A. DCO-OFDM System

Fig. 17 shows the DCO-OFDM system model. The IDFT of complex, Hermitian symmetric symbol vector $X = [X_0, X_1, ..., X_{N-1}]$, yields a real signal. Similar to the ACO-OFDM case, $N(L - 1)$ zeros are inserted in the middle of $X$ for oversampling. $X_0$ is set to be zero. Due to the large PAPR of the OFDM signal, a large DC bias $\Upsilon$ is required [6]. The time domain signal is given as follows:

$$x_{\text{DCO}}[n] = (x_{\text{BP}}[n] + \Upsilon)^+, \quad (41)$$

where $x_{\text{BP}}[n] = \frac{1}{\sqrt{LD}} \sum_{k=0}^{N-1} X_k \exp \left( \frac{j2\pi kn}{N-L} \right)$ for $n = 0, 1, 2, ..., NL - 1$. The bias $\Upsilon$ is set relative to the standard deviation of $x_{\text{BP}}[n]$ [6]:

$$\Upsilon = \mu \sqrt{\mathbb{E} \left[ \left| x_{\text{BP}}[n] \right|^2 \right]} = \mu \sigma_x, \quad (42)$$

where $\sigma_x^2$ is the average electrical power of $x_{\text{BP}}[n]$.

B. PDF of DCO-OFDM Signal

The DCO-OFDM signal has a truncated Gaussian distribution [6], given as follows:

$$f_{x_{\text{DCO}}}(x) = Q \left( \frac{\Upsilon}{\sigma_x} \right) \delta(x) + \frac{u(x)}{\sqrt{2\pi} \sigma_x} \exp \left( - \frac{(x - \Upsilon)^2}{2\sigma_x^2} \right),$$

where $Q(.)$ is the Q-function. The optical power is given by

$$P_o = \mathbb{E}[x_{\text{DCO}}] = \int_{-\infty}^{\infty} x f_{x_{\text{DCO}}}(x) dx$$

$$= \Upsilon \left( 1 - Q \left( \frac{\Upsilon}{\sigma_x} \right) \right) + \frac{\sigma_x}{\sqrt{2\pi}} \exp \left( - \frac{\Upsilon^2}{2\sigma_x^2} \right). \quad (43)$$

The electrical power allocated to individual subcarriers is

$$P_e = \frac{N}{N - 2} \sigma_x^2. \quad (44)$$

This relates $\sigma_x$, $\Upsilon$ and $P_o$. To express the achievable rate of DCO-OFDM, we will need to relate these with the electrical power per subcarrier $P_e$ and the clipping distortion. Thus we will need to a model of the clipping distortion, which is given next.

C. Modeling of Clipping Distortion

The clipping operation in DCO-OFDM introduces non-linear distortion in the signal. The effect of memory-less non-linearity on Gaussian signals is modeled using the Bussgang theorem [30]. Since the order of addition DC biasing and clipping is not significant, we consider the bipolar signal $x_{\text{BP}}(t)$, where $\overline{x} = \max \{ -\Upsilon, x \}$, for ease of analysis. The signal $\overline{x}_{\text{BP}}(t)$ is expressed as a sum of an attenuated replica of $x_{\text{BP}}(t)$ and uncorrelated distortion $d(t)$ as follows:

$$\overline{x}_{\text{BP}}(t) = \alpha x_{\text{BP}}(t) + d(t), \quad (45)$$

where $\alpha$ is correlation coefficient of $x_{\text{BP}}(t)$ and $\overline{x}_{\text{BP}}(t)$, i.e.,

$$\alpha = \mathbb{E}[x_{\text{BP}}(t)\overline{x}_{\text{BP}}(t)] = \int_{-\infty}^{\infty} x f_{x_{\text{BP}}}(x) dx = \int_{-\infty}^{-\Upsilon} -x f_{x_{\text{BP}}}(x) dx + \int_{-\Upsilon}^{\infty} x^2 f_{x_{\text{BP}}}(x) dx = Q \left( \frac{\Upsilon}{\sigma_x} \right), \quad (46)$$

which is time invariant if rectangular pulse shaping is applied on IDFT output [30]. $f_{x_{\text{BP}}}(x)$ is zero mean Gaussian PDF with variance $\sigma_x^2$. At the receiver, the DFT of $x_{\text{BP}}[n]$ leads to

$$X_{\text{BP}}[k] = \alpha X_{\text{BP}}[k] + D[k], \quad (47)$$

where $D[k] = \text{FFT}\{d[n]\}$ is the distortion term at $k$th sub-carrier, which is a sum of random variables that may not be statistically independent. However, for large $N$, simulations show that $D[k]$ can be estimated as
Gaussian with reasonable accuracy for even dependent variables [30]. Therefore, assuming the Gaussianity of $D[k]$ provides a good approximation. Now, $\sigma_2^2$ is found using (45):

$$\sigma_2^2 = \mathbb{E}[|x_{BP}|^2] - \alpha^2 \mathbb{E}[|x_{BP}|^2],$$

where $\alpha$ is given by (46), $\mathbb{E}[|x_{BP}|^2] = \sigma_x^2$ and $\mathbb{E}[|x_{BP}|^2]$ is:

$$\mathbb{E}[|x_{BP}|^2] = \int_0^\infty x^2 f_{x_{BP}}(x) dx = \frac{\gamma}{\sigma_x} \sqrt{\pi} \left( \frac{\gamma}{\sigma_x} \right) + \sigma_x^2 \left( 1 - Q \left( \frac{\gamma}{\sigma_x} \right) \right) - \frac{\gamma \sigma_x}{\sqrt{\pi}} \exp \left( - \frac{\gamma^2}{2 \sigma_x^2} \right),$$

where $f_{x_{BP}}(x) = f_{x_{DCO}}(x + T)$.

D. Achievable Rate

The received signal, after the DFT, is expressed as follows:

$$Y[k] = \alpha X_{BP}[k] + D[k] + W[k] + \sqrt{N} \delta[k].$$

The DC bias only appears at the sub-carrier $k = 0$, so it does not affect the energy of data carriers. With complex Gaussian $X_{BP}[k]$, an achievable rate per sub-carrier (sc) is given by

$$C_{DCO}^{(sc)} = \log_2 \left( 1 + \frac{\alpha^2 P_e}{\sigma_w^2 + \sigma_d^2} \right) \text{bits/subcarrier.}$$

Since every DCO-OFDM symbol comprises of $N/2 - 1$ data bearing sub-carriers, transmitted in $T_s$ seconds, information rate per second is given as follows:

$$C_{DCO}^{(sym)} = \frac{N - 2}{2T_s} \log_2 \left( 1 + \frac{\alpha^2 P_e}{\sigma_w^2 + \sigma_d^2} \right) \text{bits/sec.}$$

The clipping noise in DCO-OFDM also leaks outside the $N/2T_s$ Hz band. Therefore, following the same reasoning as used for ACO-OFDM in Section III-A6, $C_{DCO}$ becomes

$$C_{DCO} = \frac{N - 2}{2T_s B} \log_2 \left( 1 + \frac{\alpha^2 P_e}{\sigma_w^2 + \sigma_d^2} \right) \times \Pr[B_e \leq B] \text{bits/sec/Hz},$$

where $B \geq N/2T_s$ Hz. Simulation results show that the energy outside the $N/2T_s$ Hz band is negligible. Fig. 18 shows the average magnitude of DFT of DCO-OFDM for Gaussian signals. It can be seen that the out-of-band clipping noise is very small. Therefore, we can approximate $B \approx N/2T_s$ and $\Pr[B_e \leq B] \approx 1$ in (53).

The clipping noise introduces a performance floor in the system. In (51), for any $P_e$, if $\sigma_w^2$ becomes infinitesimally small, there will still be a noise energy $\sigma_d^2$. Therefore, the effective electrical SNR can never be greater than $P_e/\sigma_d^2$. Hence, as $P_e/\sigma_w$ increases, or equivalently $\sigma_w$ decrease, then $C_{DCO}$ saturates to a constant for a given $\mu$.

Fig. 19 shows the achievable rate of DCO-OFDM with varying choices of DC bias (determined by parameter $\mu$ according to (42)). The increase in DC bias has different effects at low and high SNR:

- At very low SNR, DCO-OFDM with smaller DC bias has larger achievable rate. This can be explained by noting that at low SNR, channel noise has the dominant effect, i.e., $\sigma_w^2 \gg \sigma_d^2$. For a given $P_e$, increasing the DC bias reduces $P_e$ and hence the effective SNR is reduced, which results in a lower achievable rate.

- At very high SNR, channel noise is small, so the clipping noise is dominant, i.e., $\sigma_d^2 \gg \sigma_w^2$. The presence of this clipping noise introduces a performance floor in the system. Since larger DC bias results in smaller clipping noise, so at high SNR, signals with larger DC bias have higher achievable rates.

VI. SINGLE-CARRIER PULSE AMPLITUDE MODULATION (SC-PAM)

In this section, we investigate the effects of pulse shapes on the information rate of SC-PAM, when using time-disjoint signaling. The geometric distribution [20], is chosen as the input distribution as it provides a tight lower bound on capacity.
A. SC-PAM System

An SC-PAM system is shown in Fig. 20. Input bit stream is fed to a PAM modulator whose constellation contains only real positive symbols $p_n$ with mean $P_o$. The pulse shaping filter yields the $n$th symbol $x_n(t)$ as follows:

$$x_n(t) = \begin{cases} p_n \phi(t-nT_s), & t \in [nT_s, (n+1)T_s], \\ 0 & \text{otherwise}, \end{cases}$$ \hspace{1cm} (54)

where $\phi(t)$ is a time-limited pulse of duration $T_s$, symmetric around $T_s/2$, such that $\frac{1}{T_s} \int_0^{T_s} \phi(t)dt = 1$ to satisfy the average power constraint in (2). The transmit signal is $x(t) = \sum_{n=0}^{\infty} x_n(t)$. Hence, $E[\int_{nT_s}^{(n+1)T_s} x(t)dt] = E[p_n] = P_o \forall n$ and the optical power constraint is satisfied. This is the output after correlation and sampling

$$r_n = \int_{nT_s}^{(n+1)T_s} x_n(t)\phi(t)dt + \int_{nT_s}^{(n+1)T_s} u_n(t)\phi(t)dt.$$ \hspace{1cm} (55)

Now, the average of $q_n$ and the power of noise $\nu_n$ at the output of correlator are given as follows

$$E[q_n] = P_q = P_o \Psi, \quad \sigma_q^2 = \sigma_o^2 \Psi,$$ \hspace{1cm} (56)

where $\Psi = \int_0^{T_s} \phi^2(t)dt$, provided that $\frac{1}{T_s} \int_0^{T_s} \phi(t)dt = 1$. The resulting discrete-time channel with input $p_n$ and output $r_n$ can thus be modeled as $r_n = \Psi p_n + \nu_n$.

B. Achievable Rate

In a single-carrier system, the achievable rate depends upon the input distribution and the spectral efficiency (SE) $B$ of the pulse shape used, defined as $B = \frac{1}{B/T_s}$. It can be written as

$$C_{\text{PAM}} = I_{\text{PAM}} \times B \text{ bits/sec/Hz},$$ \hspace{1cm} (57)

where $I_{\text{PAM}}$ is mutual information between the input $p_n$ and the output $r_n$ of the discrete-time channel.

1) Input Distribution: Capacity bounds for SC-PAM over a discrete-time AWGN channel were found in [18]–[20]. In [20], a geometric distribution was shown to yield a tight lower bound on the channel capacity. This lower bound is found as follows:

$$I_{\text{PAM}} = - \int f_{r_n}(x) \log_2 \left( f_{r_n}(x) \right) dx - \frac{1}{2} \log_2 \left( 2\pi e \beta^2 \right),$$ \hspace{1cm} (58)

where $\beta > 0$ is such that $I_{\text{PAM}}$ is maximized at a given SNR and $f_{r_n}$ is given as follows

$$f_{r_n}(x) = \sum_{n=0}^{\infty} \frac{1}{1 + \beta P_q / \sigma_o} \left( \frac{\beta P_q / \sigma_o}{1 + \beta P_q / \sigma_o} \right)^n \times \frac{1}{\sqrt{2\pi\beta^2}} \exp \left( -\frac{(x - q_n)^2}{2\beta^2} \right).$$ \hspace{1cm} (59)

Using channel coding techniques, this rate can be guaranteed, with an arbitrarily small error probability [29].

![Fig. 21. Pulse shapes and their spectra. Note that $\phi(t)$ must be scaled such that $\frac{1}{T_s} \int_0^{T_s} \phi(t)dt = 1$.](image)

![Fig. 20. An SC-PAM System.](image)

**TABLE II**

<table>
<thead>
<tr>
<th>Pulse Shape</th>
<th>Spectral Efficiency (1/sec/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blackman window</td>
<td>0.5525</td>
</tr>
<tr>
<td>Hann window</td>
<td>0.6471</td>
</tr>
<tr>
<td>Cosine window</td>
<td>0.6455</td>
</tr>
<tr>
<td>PSWF with THB = 1.4</td>
<td>0.7333</td>
</tr>
<tr>
<td>PSWF with THB = 1.8</td>
<td>0.6875</td>
</tr>
<tr>
<td>PSWF with THB = 2</td>
<td>0.6471</td>
</tr>
</tbody>
</table>

2) Pulse Shape: Unlike RF systems, Nyquist pulses like sinc and raised cosine, cannot be employed in IM/DD systems due to the non-negativity constraint. Instead, time-disjoint signaling using a signal space with rectangular pulse has been used in previous works [17], [18]. However, transmission of rectangular pulses are wasteful in terms of bandwidth. In order to utilize the available bandwidth efficiently, we need to use pulse shapes with higher SE. Theoretically, time-limited pulses require infinite bandwidth for distortion-less transmission. However, in a practical system, we consider the required bandwidth as that containing most of the signal energy. Here we will define SE using the FEB as follows: $B = 1/(B/T_s)$ for a given $1 - \epsilon$. Note that for SC-PAM, this is independent of the transmit symbol $p_n$.

Thus, instead of rectangular pulses, we choose to use prolater spheroidal wave functions (PSWFs) [31]. Those are considered optimal in terms of their time-frequency concentration properties. They are defined by
a given time-half bandwidth (THB) product. Here, we only consider the first, positive pulse from the set of orthogonal PSWFs. Fig. 21 shows various pulse shapes and their spectra. Their SEs for $\epsilon = 0.005$ are given in Table II.

In numerical comparison (Section VII), we will use PSWF with THB=1.4. For this pulse, $\Psi = 1.35$ which gives $P_q/\sigma_v = 1.162 P_o/\sigma_w$. This provides the necessary tools to compute the achievable rate of the SC-PAM scheme.

VII. COMPARISON OF RESULTS AND DISCUSSION

A. Simulation Parameters

In this section, we compare the achievable rates of all the OWC modulation schemes under consideration. Fig. 22 shows a comparison of achievable rates of SC-PAM, ACO-OFDM, FACO-OFDM, SEE-OFDM, FSEE-OFDM, and DCO-OFDM. The FEB in each case is such that $1 - \epsilon = 0.995$. We see that the relative performances differ at low, medium and high SNR. For SC-PAM, PSWF with THB=1.4 is used, for which $B = 0.733$ sec/Hz. For FACO-OFDM and FSEE-OFDM, $B_{LP} = N/2T_s$ Hz. The results for FSEE-OFDM are shown only for $R = 4$. For $R = 2, 3$, the achievable rates would fall between FACO-OFDM and FSEE-OFDM ($R = 4$).

B. Observations

From the results in Fig. 22, we make the following observations:

- At low SNR, SC-PAM has a superior performance, followed by DCO-OFDM with smallest $\mu$.
- DCO-OFDM improves as SNR increases due to its better spectral efficiency than SC-PAM which uses time-disjoint pulses.
- At moderate and high SNR, the effect of bandwidth efficiency becomes more pronounced. This explains the changes in the relative behavior of SC-PAM and OFDM variants.
- SEE-OFDM yields a higher achievable rate than ACO-OFDM at the cost of increased system complexity.
- Since ACO-OFDM is the least bandwidth-efficient, it has the lowest rate. FACO-OFDM nearly doubles the rate due to reduced bandwidth. However, due to the added DC-bias, the rate of FACO-OFDM is slightly less than double that of ACO-OFDM.
- Similar relationship is found between SEE-OFDM and FSEE-OFDM.
- FACO-OFDM provides a greater advantage over ACO-OFDM than SEE-OFDM does. Moreover, FACO-OFDM is also less complex to implement than SEE-OFDM, both at the transmitter and at the receiver.
- FSEE-OFDM has the best overall performance.
This is because it packs the maximum signal energy in the smallest bandwidth.

- The rate of DCO-OFDM closely follows the rate of FSEE-OFDM until its performance floor is reached.

C. Discussions

The superior performance of SC-PAM is intuitive since at low SNR, average optical power has the dominant effect. At low SNR, it is important to have the maximum optical power allocated to the information-bearing symbols. Similar reasoning explains the behavior of DCO-OFDM with different values of $\mu$. The performance of SC-PAM can be improved by using Nyquist pulses that overlap in time domain such as those recently proposed in [21]. However, time-disjoint pulses are simpler to implement.

The achievable rate of DCO-OFDM improves with SNR, which can be explained as follows: Recall that the PSWF with THB 1.4 has a spectral efficiency of $B = 0.7333$, i.e., it sends 0.7333 real-valued symbols per second and Hertz on average, whereas DCO-OFDM sends approximately almost 1 complex symbol per second and Hertz. Sinc or raised cosine pulses, with appropriate DC bias to ensure non-negativity, may also be considered for improving the bandwidth utilization of SC-PAM.

The results for FSEE-OFDM are shown for only $R = 4$ in Fig. 22. For larger $N$, $R$ can also be larger and more bandwidth can be utilized, thus some more increase in achievable rate at high SNR can be predicted. However, increasing $R$ increases the system complexity.

The comparison of the achievable rates of FSEE-OFDM and DCO-OFDM is explained as follows: The rate of DCO-OFDM is close to that of FSEE-OFDM until the performance floor of DCO-OFDM is reached. This is because DCO-OFDM uses all the sub-carriers and FSEE-OFDM uses all or most of the sub-carriers. They also have the same bandwidths and both require large DC components. However, FSEE-OFDM provides the advantage of distortionless transmission over DCO-OFDM, which incurs clipping noise. As a result, the achievable rate of DCO-OFDM does not scale well with increasing SNR. However, FSEE-OFDM is more computationally complex to implement.

VIII. Conclusion

We conducted analysis and simulations to evaluate and compare the information rates of ACO-OFDM, SEE-OFDM, DCO-OFDM and SC-PAM over IM/DD channel with optical power and bandwidth constraints. The main contribution in this work is the incorporation of effects of signal construction in information theoretic analysis, which is in contrast with previous works that focus more on the input probability distributions with no bandwidth constraint in case of multi-carrier techniques. The nature of signal construction in IM/DD systems leads to an increased requirement of bandwidth and/or power. Motivated by the observations made in the comparative study, we proposed two improved bandwidth-efficient, distortionless schemes, namely FACO-OFDM and FSEE-OFDM. These two schemes have been shown to provide a noteworthy benefit over the existing techniques in terms of spectral efficiency. FSEE-OFDM has been shown to yield the best overall performance among all the six modulation schemes under consideration.

References