# Projection Methods for Optimal Control

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**Abstract**

We consider the minimum-energy control of a car, which is modelled as a point mass sliding on the ground in a fixed direction, and so it can be mathematically described as the double integrator. The control variable, representing the acceleration or the deceleration, is constrained by simple bounds from above and below. Despite the simplicity of the problem, it is not possible to find an analytical solution to it because of the constrained control variable. To find a numerical solution to this problem we apply three different projection-type methods: (i) Dykstra's algorithm, (ii) the Douglas–Rachford (DR) method and (iii) the Aragón Artacho–Campoy (AAC) algorithm. To the knowledge of the authors, these kinds of (projection) methods have not previously been applied to continuous-time optimal control problems, which are infinite-dimensional optimization problems. The problem we study is posed in infinite-dimensional Hilbert spaces. Behaviour of the DR and AAC algorithms are explored via numerical experiments with respect to their parameters. An error analysis is also carried out numerically for a particular instance of the problem for each of the algorithms.

*Key words*: Optimal control, Dykstra projection method, Douglas-Rachford method, Aragón Artacho–Campoy algorithm, Linear quadratic optimal control, control constraints, Numerical methods.

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