PHYS 232 April 5, 2024

- Formal Report due Apr. 10 @ 11am
- Final Exam: Can bring an 8.5"×11" piece of paper with anything written on it (both sides). Bring a calculator. Graphing calculators are fine.

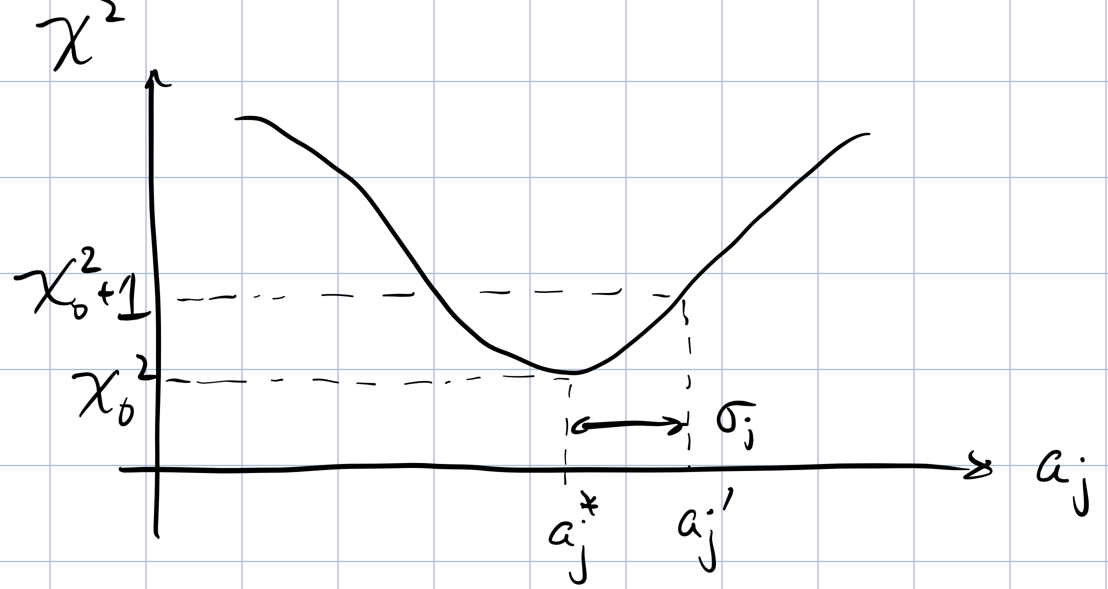
Last Time:

If meas. $(x_i, y_i \pm \sigma_i)$ for $i = 1..N$ { fit the data to a model with m parameters $a_1, a_2, \ldots, a_m$ then we have $\nu = N - m$ degrees of freedom and we expect

$$\chi^2 \approx \nu = N - m$$

The reduced chi-squared χ_ν^2 is defined to be $\chi_\nu^2 = \frac{\chi^2}{\nu}$. For a good fit, expect

$$\chi_\nu^2 \approx 1.$$



If a_j^* corresponds to minimum value of χ^2 which is denoted χ_0^2 and ...

a_j' corresponds to a value of χ^2 equal to $\chi_0^2 + 1$ (an increase of 1 above the minimum value), then ...

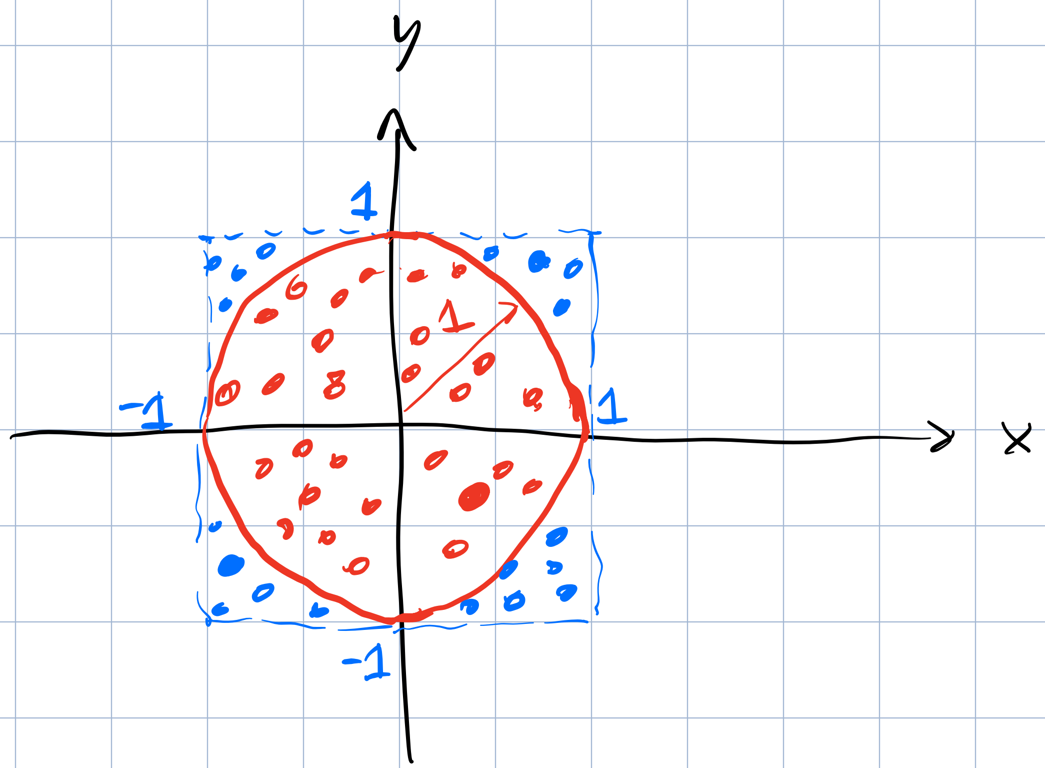
$|a_j' - a_j^*|$ is an estimate of σ_j which is the uncertainty in the best-fit value of parameter a_j.

Monte Carlo Simulations (Brief Introduction)

- useful for numerical integration (esp. high-dimension integrals)

The Monte Carlo method uses repeated random sampling to obtain numerical results.

Example 1: Computing π.



Assume that we can produce random nos. uniformly drawn from the interval $[-1, 1]$

Repeatedly draw sample pts (x_i, y_i) from $[-1,1] \times [-1,1]$

Prob. that random pt lands within the circle is

$$\mathbb{P} = \frac{\text{area of unit circle}}{\text{area of square}}$$

$$\mathbb{P} = \frac{\pi(1)^2}{2 \cdot 2} = \frac{\pi}{4}$$

$$\therefore \pi = 4\mathbb{P}$$

Have expressed desired quantity π in terms of a prob. $\mathbb{P}$. Est. value of $\mathbb{P}$ using Monte Carlo simulation.

To determine value of $\mathbb{P}$, we generate n random coordinates (x, y). Prob that we get a pt inside the circle (hit) is

$$\mathbb{P} = \frac{\#\text{ hits}}{\#\text{ trials}} = \frac{Z_n}{n}$$

no. of hits

trials.

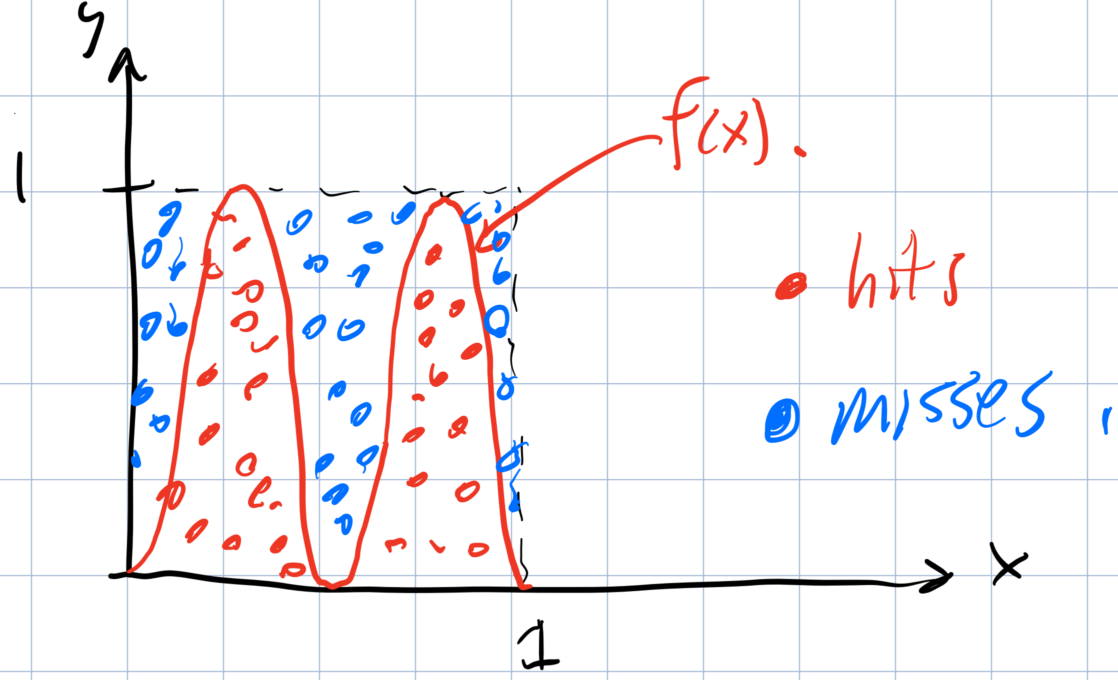
$$\mathbb{P} = \frac{Z_n}{n} = \frac{\pi}{4} \Longrightarrow \boxed{\pi = 4\frac{Z_n}{n}}$$

Hit & Miss Monte-Carlo Integration

Assume that we want to evaluate

$$\int_0^1 f(x)\,dx = \text{area under a curve.}$$

Eg. $f(x) = \frac{1}{27}\left(-65536x^8 + 262144x^7 - 409600x^6 + 311296x^5 - 114688x^4 + 16384x^3\right)$



on $0 \le x \le 1$, this fcn fits with unit square.

Use the same hit & miss simulation to find prob of a point landing beneath the curve.

$$\mathbb{P} = \frac{Z_n}{n} = \frac{\text{area under curve}}{\text{area of square}} = \frac{\int_0^1 f(x)\,dx}{1 \times 1}$$

$$\boxed{\therefore \int_0^1 f(x)\,dx = \frac{Z_n}{n}}$$

Each trial of "drop" has a prob of $\underbrace{\text{success } \mathbb{P}}_{\text{hit}}$ & prob. of $\underbrace{\text{failure } 1-\mathbb{P}}_{\text{miss.}}$

$\Rightarrow$ Binomial dist'n

Avg. no. of pts landing below the fcn is

$$Z_n = \mathbb{P} n$$

Variance $\sigma_{Z_n}^2 = n\mathbb{P}(1-\mathbb{P})$

$$\therefore \sigma_{Z_n} = \sqrt{n\mathbb{P}(1-\mathbb{P})}$$

$$\propto \sqrt{n}$$

$$I = \int f(x)\,dx \simeq \frac{Z_n}{n}$$

$$\boxed{\sigma_I = \frac{1}{n}\sigma_{Z_n} \propto \frac{1}{\sqrt{n}}}$$

The uncertainty in our est. of I decreases as $\frac{1}{\sqrt{n}}$