PHYS 232: PMT Monte Carlo Simulation

Previous analysis allowed to calc. avg. no. of electrons expected at anode for a given no. of incoming photons.

What does the distribution of the no. of electrons arriving at the anode look like? i.e. how does N vary when we repeat the same exp. over & over?

On average, get 7.5 secondary e- at each dynode (assuming that volt. require per secondary electron was 20V & volt. diff. between adjacent dynodes was 150 V).

However, don’t always get 7.5 secondary electrons per incoming e-. Sometimes get fewer, sometimes get more.
Distribution of secondary electron number should follow a Poisson distribution (counting statistics).

How does this randomness affect no. of electrons collected at anode? \( \rightarrow \) Monte Carlo simulation

Structure of Monte Carlo sim.

1. Suppose that \( N_p \) photons incident on photocathode.

   For each photon, generate a random no. \( x \) from a uniform dist'n on \([0, 1]\)

   If \( x < Q.E. (0.23) \) \( \rightarrow \) photoelectron is produced.

   \[ pe = pe + 1 \]

   After all \( N_p \) photons are considered, we know total no. of photoelectrons that
will be incident on the first dynode.

2. p.e. hit first dynode.

On average, each p.e. creates
volt. of \[ V_1 \quad \rightarrow \quad \frac{V_1}{20} \quad \text{photocathode volt.} \]
first dynode \[ \frac{V_1}{20} \quad \text{secondary electrons.} \]

.: avg. total no. of secondary electrons
at first dynode is

\[
p.e. \left( \frac{V_1}{20V} \right) = \mu \quad \text{we will use } \mu \text{ as the mean in our Poisson dist'n.}
\]

Won't always get exactly \( \mu \) secondary electrons. Actual value is determined
by Poisson dist'n:

\[
P_p(x; \mu) = \frac{\mu^x e^{-\mu}}{x!}
\]

prob. of obtaining \( x \) when the mean is \( \mu \)
Use Maple to generate random no. $N_{D1}$ drawn from a Poisson dist'n w/ mean $\mu = p.e\left(\frac{V_i}{20V}\right)$ no. of secondary electrons at first dynode.

Now total no. of electrons leaving the first & going on to the second dynode is:

$$N_{el} = p.e. + N_{D1}$$

secondary electrons generated @ dynode.
incoming electrons from photocathode

3. Repeat step 2 for all dynodes.

Eg. $\mu = N_{el}\frac{(V_2 - V_i)}{20V}$

Draw a no. from Poisson with mean $\mu$. That no. gives the secondary electrons $N_{D2}$
\[ N_{e2} = N_{e1} + N_{o2} \]

After completing for all dynodes, have the total no. of electrons arriving at anode.

4. Repeat step 1 to 3 many many times, plot a histogram of the no. of anode electrons.