Fishing in a Shallow Lake: Exploring a Classic Fishery Model in a Habitat with Shallow Lake Dynamics.

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Abstract

Renewable resources such as fish exist within an ecosystem. Harvesting activities may directly impact the habitat, beyond the influence caused by changing the balance of species. Where the relationship between the target species and the ecosystem is not monotonic, then it must be explicitly considered. A classic fisheries model is embedded in a habitat that exhibits shallow lake dynamics, where carrying capacity depends on habitat health and fishing effort damages the habitat. Numerical explorations of the model suggest that a new fishery in such a setting should be managed to protect the health of the habitat, while it may not be optimal to restore a fishery that has already degraded its habitat. Hysteresis in the habitat dynamics manifests itself as multiple steady states for dynamic and static optima, and possibly multiple open access solutions. Conventional policy tools applied in their classic form are unlikely to be effective.

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1 Introduction

Ecosystems throughout the world are influenced by human activities. In numerous situations these activities are overpowering the normal ecosystem resilience, moving them to new and less socially desirable states. These states are then found to themselves be resilient in the face of forces pushing towards restoration. Extensive surveys by Muradian [2001] and Folke et al. [2004] report that many ecosystems have multiple equilibria, pointing out that management actions can contribute to selecting which equilibria prevail [see also Rosser Jr., 2001]. In grasslands, grazing and fire control can allow the establishment of woody species, which are resistant to fire and of little use to grazers. In some forests, insect and fire control activities can lead to the accumulation of flammable biomass, a massive fire, establishment of a grassland, and very slow recovery of the forest. In some terrestrial and aquatic food webs, harvesting of one species can lead the web to reorganized, with the result that the target species is no longer present. In the most extreme cases, such as that of the kelp forest, ecosystem stability depends on a 'keystone' species and its removal leads to complete reorganization [Batabyal, 2002].

In the marine setting, beyond removing a portion of the stock, there are a variety of ways that fishing effort itself can damage the habitat. The impact of bottom dragging and shellfish dredging, which damage or destroy the substrate that supports the habitat, are quite obvious [Collie et al., 2000, presents a meta-analysis of research on bottom disturbing fishing methods]. However, fishing can change the food web structure, such that it switches to a new equilibrium in which the target species yield is much lower [Scheffer et al., 1997, Hare and Mantua, 2000]. Recognition of these impacts, and the difficulty ecosystems have in recovering, has lead to calls for more ecosystem focused management [Botsford et al., 1997], and in particular the use of marine reserves and no fish zones as a superior fisheries management tool [for example, Sumaila et al., 2000, Lindholm et al., 2001].

In this paper, we couple the classic fisheries model with a habitat model that has a fold catastrophe, the now well known shallow lake model [Carpenter and Ludwig, 1999, Brock and De Zeeuw, 2002, Brock and Starrett, 2003, Mäler et al., 2003]. With this addition, the model exhibits hysteresis, and can become 'stuck' in an undesirable equilibrium. The properties of the shallow lake model where extensively examined by Brock and Starrett [2003] and Wagener [2003], who among other things examined the potential for multiple equilibria and the role of Skiba [Skiba, 1978] points. The current analysis shares much with Crépin [2007], who considers processes with different time scales. As in her work, we find that the interesting results follow from considering multiple states, but that this means there are few general statements that can be made. In a discussion of the possibility of multiple equilibria in well behaved systems, Wirl [2004] also comments on the difficulty of identifying thresholds in more complex systems.

A number of investigations have modelled the interaction between a fishery and the habitat that it inhabits. Barbier and Strand [1998], examined the relationship between mangrove deforestation and the shrimp fishery in Campeche, Mexico. A more recent application of their model [Barbier, 2003] endogenizes the mangrove deforestation process. In Knowler et al. [2002], the authors examine the impact of nutrient loading on the Black Sea anchovy fishery. Nutrient loading is beneficial at low levels, contributing to an increase in the food consumed by the fish. However, high nutrient levels can lead to a regime shift to a less productive equilibrium. Nutrient issues are important in shallow lake systems, where the lake responds to phosphorus inputs by switching between two regimes, the desirable oligotrophic and undesirable eutrophic conditions [Carpenter and Ludwig, 1999]. Once the lake becomes stuck in a eutrophic state, phosphorus inputs must be lowered far below the switch level before it will switch back. Knowler et al. [2002] suggest that a similar effect may exist for fish stocks, such that a fishery which is over fished may need to have catch held far below the equilibrium level to allow the system to recover. This is consistent with the shallow lake dynamics, but unlikely to be a direct influence of nutrient load on fish stock health [Willemsen, 1980]. While our emphasis is on the ecosystem supporting the fishery, Guttormsen et al. [2008] showed that results like those of Knowler et al. [2002] can result from artificial selection pressures, and Sterner [2007] found that unobserved sub-population structure can also result in a fishery getting stuck in a low productivity situation. Results similar to ours are found for a stock pollution model by Tahvonen and Salo [1996], particularly that non-convexities in either the objective function or state transition function can result in multiple equilibria. The idea that actions can affect the habitat carrying capacity is suggested by Armstrong [2006] as a way to model the benefits of marine reserves, where it is argued that the removal of fishing effort in the reserve results in a higher carrying capacity.

2 Model

The habitat dynamics are based on the the shallow lake system (now often called shallow lake dynamics, SLD) described by Carpenter and Ludwig [1999]. The dynamic behaviour of this system has been extensively studied by Brock and Starrett [2003] and by Wagener [2003], with particular emphasis on the multiple equilibrium situations that can exist. We modify the interpretation, in that where their state variable is phosphorus load, ours is a more generic 'ecosystem damage'. As in Barbier [2000], greater ecosystem health contributes to greater harvest. Absent outside forcers such as fishing effort, the ecosystem naturally reverts to a no damage state. To this, we add a logistic growth fish species, with the modification that the carrying capacity of the habitat is a decreasing function of ecosystem damage. Where shallow lake analyses typically consider phosphorus input as the control, in our analysis fishing effort serves this purpose. We focus on the case where fishing effort both contributes to catch and negatively impacts on ecosystem health. The nonconvex shallow lake dynamics of habitat damage and fishing effort precludes the use of a single state variable. We proceed by building the habitat dynamics into a classic fisheries model [see Clark, 1976, for comprehensive treatment of the classic model] through a habitat health dependent carrying capacity.

2.1 General Form

We begin with a general model, without any specific functional forms. This model integrates three key elements, the dynamics of the habitat state, the dynamics of the fish stock, and economic behaviour. The habitat dynamics are represented as

$$\dot{z} = -r(z) + d(y, z)$$

where z measures habitat damage, taking on the value of zero for a health habitat. The habitat naturally recovers at a rate given by $r(z)$, where $r(z)$ is positive. No restrictions are put on the higher derivatives, other than their existence. Fishing effort y causes damage $d(y, z)$ to the habitat. Damage is assumed to be nonnegative - no level of fishing effort can repair the habitat, regardless of how damaged it is. The exact form of $d(y, z)$ will depend on the specific habitat and type of fishing gear. In general damage is expected to increase in effort. However, that could be at an increasing or decreasing rate. Likewise, the response to z could also be increasing or decreasing, depending on whether there are thresholds above or below which the habitat is more or less sensitive to damage.

The fish stock dynamics are given by

$$\dot{x} = g(x, z) - h(x, y)$$

where stock x instantaneously increases by $g(x, z)$. Growth is positive for some range of x, with some size K (carrying capacity) above which growth is negative. Habitat health can affect the stock by changing the growth rate, carrying capacity, minimum viable population, or some combination thereof. Harvesting activity $h(x, y)$ reduces stock growth. Harvest size is increasing in both the size of the stock and the amount of effort. It is assumed that when $\dot{x} = 0$, the fish stock dynamics can be solved to yield single valued relationships $y = y_{ss}(x, z)$ and $x = x_{ss}(y, z)$.

Willingness to pay for a unit of harvest is $p(h)$, which is decreasing in h, while effort cost is $c(y)$. Assuming that there is a single market price for harvested fish, instantaneous industry profit is therefore

$$p(h(x, y))h(x, y) - c(y)$$

while consumer surplus is

$$\int_0^{h(x,y)} p(\tau)d\tau - c(y)$$

Relying on the existence of the relationship $y = y_{ss}(x, z)$ allows some aspects of the different solutions to be characterized. In particular, two partial solution sets can be identified in $x \times z$ space. Solving $\dot{z} = 0$ yields

$$d(y_{ss}(x, z), z) = r(z)$$

which implicitly defines points where the physical system is in steady state, absent any economic content. Different economic solutions will choose among these steady state points. In addition to this first partial solution, the $\dot{x} = 0$ result can be combined with different economic solution concepts to identify a set of partial steady state points where the economic condition is satisfied. The intersection of these two sets of points identifies the steady states for the combined system.

In what follows we will build towards representing the behaviour of the system in the space of the two state variables. The relationship between x and z that must hold when the physical system is in steady state will be call the *response function*. By isolating an expression for the economic solution condition and imposing a subset of steady state conditions, we define a *conditional first order condition function* or *conditional FOC*. Where the conditional FOC intersects with the response function identifies that subset of steady states for the physical system which also satisfy economic solution concepts.

2.1.1 Open Access

Setting instantaneous profit to zero identifies possible open access solutions. Imposing $\dot{x} = 0$ allows substitution for h and y such that

$$p(g(x,z))g(x,z) = c(y_{ss}(x,z)) \tag{1}$$

identifies combinations of x and z where profit is zero. Open access solutions for the system occur where this relationship intersects with the response function.

2.1.2 Static Optimum

In addition to the above assumptions, we also assume that there exists a relationship $z = z_{ss}(y)$ for the following exposition. Maximizing steady state consumer surplus then involves solving

$$\max \int_0^{h(x_{ss}(y,z_{ss}(y)),y)} p(\tau)d\tau - c(y)$$

the first order condition for which is

$$p(g)[h_y + h_x x_{ss,y} + h_x x_{ss,z} z_{ss,y}] = c_y$$

where arguments have been dropped and subscripts involving x, y or z indicate partial derivatives in those arguments. The term $h_y + h_x x_{ss,y} + h_x x_{ss,z} z_{ss,y}$ captures the conventional textbook 'static equilibirum' treatment, with the additional constraint that two steady state conditions are being maintained at the same time. The first term captures the impact on harvest of a change in effort, while the second and third terms are adjustments reflecting the constraint that the solution must be a steady state in x and z. The impact of y on z is transmitted through the impact of z on x, because the habitat state does not enter directly into the profit function. Note that we are not assuming that $z = z_{ss}(y)$ is unique, leaving open the possibility of multiple solutions to this first order condition.

2.1.3 Dynamic Optimum

We maximize the present value of consumer surplus by using the Hamiltonian. The Hamiltonian is

$$H = \int_0^{h(x,y)} p(\tau)d\tau - c(y) + \pi_x[g(x,z) - h(x,y)] + \pi_z[-r(z) + d(y,z)]$$

with first order conditions

$$\begin{aligned} p(h)h_y - c_y - \pi_x h_y + \pi_z d_y &= \partial H/\partial y &= 0 \\ p(h)h_x + \pi_x(g_x - h_x) &= \partial H/\partial x &= \rho\pi_x - \dot{\pi}_x \\ \pi_x g_z + \pi_z(-r_z + d_z) &= \partial H/\partial z &= \rho\pi_z - \dot{\pi}_z \end{aligned}$$

together with the equations of motion for x and z. Assuming that $\partial H/\partial y$ allows a solution $y_{FOC} = y(x, z, \pi_x, \pi_z)$ to be identified, the equations of motion for the system are

$$\begin{aligned} \dot{x} &= g(x,z) - h(x, y_{FOC}) \\ \dot{z} &= -r(z) + d(y_{FOC}, z) \\ \dot{\pi}_x &= \pi_x\left[\rho - (g_x - h_x)\right] - p(h)h_x \\ \dot{\pi}_z &= \pi_z\left[\rho - (-r_z + d_z)\right] - \pi_x g_z \end{aligned}$$

where arguments have been dropped for $\dot{\pi}_x$ and $\dot{\pi}_z$ expressions to simplify the presentation. Interpretation of $\dot{x}$ and $\dot{z}$ has already been done. The last two equations can be rewritten in their Hotelling rule form.

Beginning first with the shadow value of the stock,

$$\rho = \frac{\dot{\pi}_x}{\pi_x} + \frac{p(h)h_x}{\pi_x} + (g_x - h_x)$$

Along the optimal path, the sum of the rate of growth of the stock shadow value, the harvest value of the stock externality, and the net growth impact of the stock, is equal to the discount rate. The shadow value can increase or decrease along this path, depending on the relative size of the other two terms on the right hand side. The path converges on a steady state where

$$\pi_x = p(h)h_x / [\rho - (g_x - h_x)]$$

At a steady state, the shadow value of the stock is equal to the present value of the harvest value of the stock externality, where the discount rate includes an adjustment for stock growth effects. Absent a stock externality ($h_x = 0$), the textbook result that the rate of growth equals the discount rate prevails (multiply through by $\rho - (g_x - h_x)$).

Turning to the shadow value of habitat health, in Hotelling form this becomes

$$\rho = \frac{\dot{\pi}_z}{\pi_z} + \frac{\pi_x g_z}{\pi_z} + (-r_z + d_z)$$

Along the optimal path, the sum of the rate of growth of the shadow value of habitat damage, the relative value of the marginal impact of habitat damage on stock growth, and the net habitat recovery impact of damage equals the discount rate. Since habitat damage is only valued through its impact on stock growth, the driver for the shadow value of habitat damage is the term $\pi_x g_z / \pi_z$. Were g_z zero, then the only reasonable value for π_z would also be zero. In steady state, the shadow value of habitat damage is

$$\pi_z = \pi_x g_z / [\rho - (-r_z + d_z)]$$

which again reflects the fact that the value of habitat damage is purely a function of its importance to stock growth, and would be zero if $g_z = 0$. With $g_z \neq 0$, the shadow value in steady state is the present value of the impact on the stock of a change in damage, with the discount rate adjusted for the habitat damage impact on habitat damage evolution. Notice that for certain specifications of $g(x,z)$, $h(x,y)$, $r(z)$ and $d(y,z)$, there will be ranges where π_x and/or π_z are undefined.

To close things up, one well known result is that the dynamic equilibrium converges on the static equilibrium as $\rho \to 0$. Setting $\rho = 0$, $\partial H/\partial y$ can be rewritten as

$$p(h)\left[h_y + \frac{h_x h_y}{(g_x - h_x)} + \frac{h_x g_z d_y}{(g_x - h_x)(r_z - d_z)}\right] = c_y$$

Using the total derivative of $\dot{z} = 0$ it is quick to show that

$$dz/dy = d_y/(r_z - d_z)$$

and with $\dot{x} = 0$, that

$$dx/dy = h_y/(g_x - h_x) + (dz/dy)g_z/(g_x - h_x)$$

With these results, it follows that

$$\partial H/\partial y = p(h)[h_y + h_x(dx/dy)] \;=\; p(h)[h_y + h_x x_{ss,y} + h_x x_{ss,z} z_{ss,y}] = c_y$$

which is the desired result. Another well known result is that as $\rho \to \infty$, the dynamic optimum solution converges on the open access solution. As $\rho \to \infty$, $\partial H/\partial y$ converges to a solution of the form $p(h)h_y = c_y$. This is not identical to the open access solution, a consequence of the fact that the optimization maximizes consumer surplus, not profit.

2.2 Specific Form

To demonstrate further results specific functional forms are selected, with the choice of $\dot{z}$ being the most critical. The shallow lake model introduced by Carpenter and Ludwig [1999] is

$$\dot{P} = l - sP + \frac{rP^q}{m^q + P^q} \tag{2}$$

(equation 1, page 753) where P is phosphorus load in the lake, l is input, sP is natural removals, and $rP^q/(m^q + P^q)$, with $q \geq 2$, measures the amount of P recycled in the lake system. The model can have three steady states, one oligotrophic, one eutrophic, and an unstable one with a phosphorus level between the other two. The recycling component captures the fact that phosphorus can accumulate in lake sediments, or be stored in lake biota, and remain captured in cycles that prevent it from being flushed out. Recognizing the conflict between those who enjoy an oligotrophic lake and those that gain by contributing to phosphorus input, together with the fact that there are often open access elements in lake management, lead to a number of subsequent economic analyses [Brock and De Zeeuw, 2002, Brock and Starrett, 2003, Mäler et al., 2003, Wagener, 2003]. For the single state shallow lake model, Brock and Starrett [2003] comprehensively discusses the existence and properties of single and multiple equilibria, and Wagener [2003] effectively deals with the relationship between parameter values and the existence of multiple equilibria and Skiba points.

We see the shallow lake model as a convenient point of departure to understand how complex ecosystem dynamics can feed back through a fishery. Instead of phosphorus, we use a state variable z to represent ecosystem health, where a healthy ecosystem has $z = 0$ and health deteriorates as z becomes larger. See Rosser Jr. [2001] for some discussion of the use of aggregate measures of ecosystem function in multiple equilibrium models. If effort induced damage is considered a disturbance of a natural succession process, then, z can be considered an 'index of succession', reaching a value of $z = 0$ for a climax community. As described in Odum [1969], succession is a complex process with forces pushing it forward and back [see also Christensen and Pauly, 1998]. The catastrophic form of the dynamics captures the existence of a phase along the evolution of this aquatic ecosystem where progress towards the climax community is very slow.

For the analysis that follows, we use the Carpenter model, with $q = 2$, so that $r(z) = \alpha z - \theta z^2/(\phi^2 + z^2)$. To ease the identification of numerical solutions, we choose $d(y, z) = my$. More complex damage functions are certainly possible and more realistic, but will greatly complicate the algebra. The key qualitative results will not be affected by this choice. The dynamics of ecosystem health are then

$$\dot{z} = mz - \alpha z + \theta z^2/(\phi^2 + z^2) \tag{3}$$

For some parametrizations, this form generates multiple equilibria. Models of this form are said to have a fold catastrophe [Zeeman, 1977], exhibiting multiple steady states and having critical points where the system switches between them. Bifurcation analysis focuses on how system behaviour evolves around steady states when parameter values are changed, with particular emphasis on the jumps to multiple steady states. Wagener [2003] explores in detail the properties of the shallow lake model from this perspective.

The classic logistic relationship would have $g(x,z) - h(x,y) = \gamma x(1 - x/K) - qxy$. One way to introduce a habitat health impact would be to have $\gamma = \gamma(z)$, with $\gamma_z < 0$. In this form, a more damaged habitat causes a slower growth of the fish stock. A second approach is to have $K = K(z)$, with $K_z < 0$. With this formulation, the more damaged the habitat, the smaller the stock it can support. Contingent carrying capacities are found in recent work on global human carrying capacity [for examples, see Meyer and Ausubel, 1999, Hopfenberg, 2003], which use $K(t)$, with t as time. A recent marine reserve model [Armstrong, 2006] uses $K(m) = K + g(m)$ where m is the relative size of the reserve. Notice also that the growth rate $\dot{x}/x$ decreases with increasing z, for all $x > 0$. A simple implementation of this approach is for $K(z) = K/(z+1)$, which can be further simplified to $K(z) = 1/(z+1)$ if we parameterize the system such that $K = 1$. With these assumptions, evolution of the stock is governed by

$$\dot{x} = \gamma x[1 - x(z+1)] - qxy \tag{4}$$

The term qxy is the normal Shaefer stock dependent effort productivity function for a fishery, with q measuring the catchability of the fish stock and x the size of the stock.

At the steady state, a relationship between x and z is immediately apparent. Imposing $\dot{x} = 0$ and simplifying, it follows that

$$y_{ss} = \gamma \left(1 - x(z+1)\right)/q \tag{5}$$

Inserting this into 3, with $\dot{z} = 0$, results in an implicit relationship defining a curve in $x \times z$ space,

$$m\gamma \left(1 - x(z+1)\right)/q = \alpha z - \theta z^2/(z^2 + \phi^2) \tag{6}$$

Making these algebraic assumptions results in relatively simple relationships that define all feasible steady state combinations of stock size, habitat health, and effort level. The different economic solutions consist of choosing a point or points along the curve represented by 6.

The implicit relationship set out in equation 6 can be rearranged to isolate a response function, defining x as a function of z.

$$x(z) = \frac{1}{z+1} + \frac{q/mr}{z+1}\left[\theta\left(\frac{z^2}{z^2+\phi^2}\right) - \alpha z\right]$$

The shape of this response function plays an important role in determining whether multiple equilibria can exist. When the habitat is perfectly healthy, $z = 0$, then stock is at the no damage carrying capacity of $x = 1$. In contrast, when $z \to \infty$, the stock converges on $x = -\alpha q/mr$. As this is not feasible, there must be at least one steady state where $x = 0$. The curvature of this relation determines if there will be more than one.

Finally, providing economic content requires specifying $p(h)$ and $c(y)$. To keep things simple, assume a downward sloping linear demand curve $p(h) = A - Bh$ and a constant marginal cost of effort cost curve $c(y) = cy$, with no fixed cost. The downward sloping demand curve is a slight departure from the more normal practise of using a constant price. Doing so provides for a positive surplus at the open access solution, and a more appealing analysis of supply and demand curves.

2.2.1 Open Access

With the specifications outlined above, the open access solution condition (equation 1) is

$$\{A - B\gamma x\left[1 - x(z+1)\right]\}\gamma x\left[1 - x(z+1)\right] = c\left[\gamma\left(1 - x(z+1)\right)/q\right]$$

With some rearrangement, z can be isolated, yielding

$$z = -(\gamma B x^3 - \gamma B x^2 + Ax - c/q)/\gamma B x^3$$

For this relationship, when $x \to 0$, $z \to \infty$, while when $x = 1$, $z = -(A - c/q)/\gamma B$. Assuming that the parameter values result in a monotonic function on $0 \leq x \leq 1$, and that fishing is sufficiently profitable $(A > c/q)$, then together with the conditions outlined above, there must be at least one open access solution.

2.2.2 Static and Dynamic Optimum

It has been shown that the static optimum corresponds to the dynamic optimum where the discount factor is zero. Thus, we focus only on the algebra of the dynamic optima. The static optimum is then located by setting $\rho = 0$ and choosing the steady state with the largest instantaneous return.

Using the definitions for $g(x, z)$, $h(x, y)$, $r(z)$ and $d(y, z)$, and assuming an interior solution, the first order condition $\partial H/\partial y$ can be quickly rearranged to yield

$$y(x, z, \pi_x, \pi_z) = \frac{A - \pi_x}{Bqx} - \frac{c - m\pi_z}{Bq^2x^2}$$

which is convenient for setting up the dynamic system. An increase in A - an upward shift of the demand curve - increases effort, while an increase in effort cost c reduces it. The effect of increasing q or B is ambiguous, depending on the size of y relative to $c - m\pi_x/Bq^2x^2$. Notice that as $B \to 0$, $y \to \pm\infty$, depending on the relative size of the two terms. For the presentation below, define $y_{FOC} = y(x, z, \pi_x, \pi_z)$.

Given the specification set out, the equations of motion for the system resolve to

$$\begin{aligned}
\dot{x} &= \gamma x[1 - x(z + 1)] - qxy_{FOC} \\
\dot{z} &= my_{FOC} - \alpha z + \theta z^2/(\phi^2 + z^2) \\
\dot{\pi}_x &= \pi_x \{\rho - [\gamma - \gamma 2x(z + 1) - qy_{FOC}]\} - (A - Bqxy_{FOC})\, qy_{FOC} \\
\dot{\pi}_z &= \pi_z \left\{\rho - \left[-\alpha + 2\theta z\phi^2/(\phi^2 + z^2)^2\right]\right\} - \pi_x(\gamma x^2)
\end{aligned}$$

These equations define the dynamics of the system along optimal paths. The steady states have already been explored for the two state variables. For the costates, in steady state

$$\begin{aligned}
\pi_x &= \{(A - Bqxy_{FOC})\, qy_{FOC}\} \{\rho - [\gamma - \gamma 2x(z + 1) - qy_{FOC}]\}^{-1} \\
\pi_z &= \pi_x(\gamma x^2) \left\{\rho - \left[-\alpha + 2\theta z\phi^2/(\phi^2 + z^2)^2\right]\right\}^{-1}
\end{aligned}$$

With some tedious manipulation, these solutions can be used to eliminate π_x and π_z from the equations. Interpretation is as for the general functional form shown above.

A particular issue that follows from the specification is the possibility of relationships between parameter values where solutions do not exist. Focusing first on π_x, it will be undefined when $\rho = \gamma - \gamma 2x(z+1) - qy_{FOC}$. Using $\dot{x} = 0$ to eliminate y_{FOC} leads to π_x being undefined when $\rho + \gamma x(z+1) = 0$. Since this can only occur for infeasible values of x or z, singular values for π_x are not an issue. Turning to π_z, it is undefined if $\rho = -\alpha + 2\theta z\phi^2/(\phi^2 + z^2)^2$. This expression depends only on parameter values and z. This identity resolves to a fourth order polynomial in z, with rather complicated expressions for the roots. There are therefore as many as four values of z, four real roots, where π_z is undefined. In the neighbourhood of these values, π_z and any functions of π_z are likely to exhibit stark changes.

To provide a compact illustration of the behaviour of this system, we solve for the response function, which relates the state variables, and a conditional first order condition function. This latter solves $\partial H/\partial y = 0$, conditional on $\dot{x} = 0$, $\dot{\pi}_x = 0$ and $\dot{\pi}_z = 0$. The manipulations themselves are left to an appendix. The result is a fifth order polynomial in x, with coefficients that include terms up to the sixth order in z. There is little practical insight that can be generated from the examination of these equations, and therefore we turn to a numerical example.

One additional result that follows from the specification occurs when $\rho \to \infty$. It follows immediately from the steady state definitions of π_x and π_z that these must both go to zero as $\rho \to \infty$. It therefore follows that in steady state $y = (Aqx - c)/Bq^2x^2$, conditional on $\dot{\pi}_x = 0$ and $\dot{\pi}_z = 0$. Setting $\dot{x} = 0$ requires $\gamma[1 - x(z+1)] = qy$. Substituting for y and rearranging to isolate z yields

$$z = -(\gamma Bx^3 - \gamma Bx^2 + Ax - c/q)/\gamma Bx^3$$

which is precisely the relationship found for the open access situation. Thus, for this specification, the conditional FOC for the dynamic optimization converges on the condition defining the open access solution.

3 Numerical Example

We choose a default case to illustrate the existence of multiple steady states, and then vary the individual parameters to demonstrate how the properties of these steady states are changed. The shallow lake model can be very sensitive to parameter choices [Scheffer et al., 1997, 2000]. The default parameter values are shown in table 1. Calculations were conducted using Octave [Eaton et al., 2008] with algorithm checking and plotting of results done with R [R Development Core Team, 2008].

[Table 1 about here.]

Before considering the impact of varying parameters, we examine in detail the default case. Figure 1 presents six perspectives on the base case. The first perspective, depicted in panel (a), shows the traditional growth curve, as modified by the introduction of a habitat damage state with hysteresis. The nonconvexity in the habitat response generates a nonconvexity in the growth function. There may be up to four different stock levels corresponding to a given harvest level. Two points are identified that satisfy the open access condition (OA), one with a positive harvest and the second at zero stock and zero harvest. The static optimum (SO) has the highest stock level of all the solutions. There are five marked points that satisfy the first order conditions for the dynamic optimum (DO), with that one which generates the largest consumer surplus having a slightly higher harvest and lower stock level than the static optimum.

[Figure 1 about here.]

Panel (b) translates the growth function into surplus, total revenue, and total cost as a function of effort. The open access solutions occur where theory says they must, namely where total revenue is equal to total costs. The total revenue curve deviates from the textbook case, in that it has a flat, slightly depressed, top. This is a consequence of using a downward sloping demand curve in place of the conventional constant price used in the textbook treatment. If difference between the revenue and cost curves is the focus, then the lowest effort point satisfying the dynamic optimum steady state conditions maximizes profit, a consequence of the fact that when harvest is low, price is high. However, the static and dynamic optima are solved for by maximizing aggregate economic surplus, which is consumer surplus plus revenue less cost. When surplus is plotted, then the surplus maximizing steady state is easy to identify. Careful inspection also shows that at the static optimum solution, the slope of the surplus curve is not equal to the cost curve, an implication of the habitat damage dynamics. A consequence of this is that there are cases with parameter values where the steady state instantaneous surplus is greater than that for the classically identified static optimum.

For the specifications used, if the system is already at the open access solution, a single effort tax set at the optimal level may not lead it to that optimum. Rotating the cost curve counterclockwise around the origin will not lead to a single intersection on the upper part of the revenue curve until the tax is so high that its intersection is to the left of the static optimum. If it is not rotated this far, then there will be multiple equilibria, and without a process for the evolution of the fleet [see Clark, 1976, Bjorndal and Conrad, 1987, Barbier and Strand, 1998, Sanchirico and Wilen, 2001, Knowler et al., 2002, for examples], it is unclear to which solution the system will converge, and even which is optimal. An alternative policy that is suggested by the graph is a lump sum tax levied on the industry, together with an effort subsidy. By shifting up the intercept of the total cost curve, and simultaneously lowering the slope, this tax regime can pick out the preferred optimum on the upper portion of the revenue curve while bypassing the indented portion.

Panel (c) explains many of the results outlined for the two classic figures of panel (a) and (b). The response function identifies all those combinations of x and z such that the system is at a biophysical steady state. The inverted 'S' shape of this curve is similar to the shallow lake phosphorus dynamics of Carpenter and Ludwig, except that their plots relate the control variable with the state variable. The three FOC curves represent the conditional first order conditions, where $\partial H/\partial y = 0$ is satisfied together with the steady state conditions $\dot{\pi}_x = 0$, $\dot{\pi}_z = 0$ and $\dot{x} = 0$. See appendix A for a similar exposition using the classic fisheries model without a habitat damage state. The response curve intersects the dynamic optimum (DO) conditional FOC curves at five points, mapping directly to the five points identified in panel (a). Reducing the discount rate to zero locates the conditional FOC curves corresponding to the static optimum. There are still five intersections. However, with zero discount rate, it is always optimal to move to that steady state which maximizes surplus, as the present value of being at this point will always outweigh any costs of moving to this point. Thus, only this intersection is identified. The curve identified as the open access FOC is not precisely a first order condition, but rather the curve identifying combinations of x and z such that profit is zero and $\dot{x} = 0$. As the discount rate approaches infinity, the conditional FOC curve converges on this curve.

The locations of the conditional FOC curves is driven in part by the singularities in the definition of the steady state for π_z. For $\rho > 0.12951906$ (approximately), there is only one conditional FOC curve, within the range of x and z considered. Values of x and z located to the left of this curve result in negative conditional

FOC values, while the converse occurs on the right side. As ρ is reduced, a line appears along which the conditional FOC goes to $-\infty$ to the left of the conditional FOC curve, and to $+\infty$ on the right. Further reducing ρ creates two lines, one above and one below the single line. The conditional FOC curve crosses these lines at two points, continuous with the curve coming from above and below. However, the conditional FOC values away from the crossing points switch as the lines are crossed, jumping to $+\infty$ on the left and falling to $-\infty$ on the right. On the left, the conditional FOC values fall rapidly to again become negative, creating the 'island' on the left side of the figure. On the right, the conditional FOC values climb rapidly and become positive again, in a way that pulls the FOC curve to the right. The fact that these singularities fall along lines is a consequence of the linear impact function. However, the potential for singularities in the steady state definition of π_z is not strictly a consequence of this specification.

Panel (d) plots harvest as a function of price. This curve was located by solving for the steady states of the system while vertically shifting the demand curve (changing A). The classic textbook result that the static optimum supply curve is upward sloping and the open access supply curve bends backwards is present, although with some variation. Focusing on the open access curve alone, the variation is the large indentation that corresponds to the 'bite' taken out of the growth and revenue functions of panels (a) and (b). From this alone, it is clear that a simple harvest tax is not able to restore the fishery to the static optimum. Shifting down the demand curve so that it intersects the open access supply curve at the optimal harvest level leads to three intersections between the supply curve and the demand curve. Without further assumptions about the dynamics of the fleet adjustment, it is not possible to determine either whether the static optimum (or maximum steady state surplus dynamic optimum) can be attained, or if it is in fact optimal to do so. However, as for effort controls, a more complex movement of the demand curve may be an effective way to manage the fishery. A harvest tax, the level of which falls as total harvest increases would effectively flatten the demand curve as well as shifting it down. Chosen correctly, this could pick out the desired harvest level on the lower portion of the open access supply curve, while eliminating the possibility of multiple equilibria. Clearly a harvest quota would also not work if set at the final target harvest level. Rather, a period of low or zero harvest would be required, of sufficient duration to allow the stock and habitat time to recover.

The most unusual curve in panel (d) is that which maps the steady states for the dynamic optima as the demand curve is shifted upwards. Classic theoretical analysis show that as $\rho \to 0$, the dynamic optima converges on the static optima, and as $\rho \to \infty$ the dynamic optima converges on the open access result. Both of these results hold for the present model. However, with this model what lies between the two extremes only weakly resembles either. For the classic model, there is a smooth transition from the static optima to the open access solution as ρ is increased. With the present model, there are two segments of the demand curve that morph and move as the discount rate increases. Specifically, the centre segment that rises from the bottom axis shrinks down and then disappears, while the segment coming down from the top of the figure - which does join the rightmost portion rising up - shrinks and shifts to become the open access supply curve. These components are present for the $\rho = 0$ case, but have been left out of the figure both to keep it clean and in recognition of the fact that when $\rho = 0$, it is always optimal to move to that steady state with the largest instantaneous payoff.

Panels (e) and (f) illustrate some of the dynamics of the system. The system is severely ill conditioned for most of the parameter values being explored, with the ratio of the modulus of the maximum and minimum eigenvalues of the characteristic matrix commonly being close to ten. This results in a strongly dominant ray for the dynamics of the system, where almost all saddle paths closely follow this ray for much of their convergence to the steady state. In panels (e) and (f), saddle paths drawn which do not originate at (1,0) are representative of thousands of such paths that were generated from points of the form $(x_{ss}, z_{ss}, \pi_{x,ss} + \Delta_x, \pi_{z,ss} + \Delta_z)$, with $\Delta_x^2 + \Delta_z^2 = \epsilon$, for small epsilon, solving backwards (reverse shooting). Paths that originate at (1,0) were also located by reverse shooting, using an implementation of an algorithm outlined in Atolia and Buffie [2007]. Unfortunately the computational burden was sufficiently large that it was not practical to locate all paths to all the steady states and thereby identify Skiba surfaces [Skiba, 1978] or separatrices [Crépin, 2007] for the system.

Panel (e) plots representative dominant axis saddle paths for each of the five steady states that exist in the default case. The points map directly from panel (c). While the saddle paths in the four dimensional space of this system do not cross, the two dimensional projections can, and in the figure clearly do. This presents a dilemma for management of this resource, as typically the only observables are physical variables, and often not precisely. In this example the optimal path involves moving in the direction of increasing damage and depleting the stock, before then reducing damage and building up the stock in a direction almost exactly opposite to that previously pursued. Moving the system along a saddle path will be difficult without precise control over the control variable, harvest effort. Indirect tools such as harvest or effort controls, that are not directly related to the levels of the variables of interest may be imprecise and even ineffective.

Panel (f) illustrates that the location of the bundle of saddle paths converging on the dominant ray can be highly sensitive to the parametrization of the system. For this figure, the rate of recovery of the habitat, α, is varied. Only saddle paths leading to the instantaneous surplus maximizing steady state are shown. At the low end of the range for α, the saddle path bundle begins in the upper left of the panel, and follows an oscillatory pattern of increasing frequency until finally converging on the steady state. Initially, increasing α reduces the amount of oscillation before convergence. Further increasing α then leads to the saddle path bundle originating in the lower right of the panel. Again increasing α results in the saddle path bundle rotating in a counterclockwise direction and collapsing the oscillations. Finally with the oscillation gone, the bundle again shifts towards the right.

Saddle paths were located for a much narrower range of parameter values. Two are shown. These paths are almost identical in the neighbourhood of an undisturbed fishery. However, when they diverge, one path leads to increasing damage and declining stock in a direction that is almost exactly opposite to the other. If only habitat state and stock level can be monitored, then if the details of the biophysical system are not know precisely, it may be difficult to manage such a fishery and lead it along the optimal path. Given the complexity of the saddle paths, management of such a system may be better done by direct measurement of the state variables. Policies such as taxes and subsidies directed at these variables may be more effective, such as suggested by Holland and Schnier [2006].

Figure 2 shows how the system steady states evolve for changes in three of the parameters that are present in the model without a habitat state. Panel (a) show the impact of changing the discount rate on the locations of steady states in $x \times z$ space. First note that the response function is unaffected. This follows from the fact that the response function simultaneously solves $\dot{x} = 0$ and $\dot{z} = 0$, which does not have any relationship with ρ. The $\rho = 0$ conditional FOC reproduces that which identifies the static optimum. As ρ is increased, the conditional FOC contracts towards the left and that part of the conditional FOC extending inward from the left shrinks. This shrinkage eliminates two of the points that satisfy the FOC for a steady state of the system.

[Figure 2 about here.]

In panel (b), the level of habitat damage for each steady state over a range of parameters is plotted, for those steady states where x, y and z are nonnegative. For each steady state, the values for the analytical Jacobian are generated and the eigenvalues of this matrix calculated. If there are any imaginary roots, which suggest there may be some degree of cycling along the saddle paths, then the marker is grey. That steady state with the highest instantaneous profit is marked by a larger dot (evident as a thicker line). At low values of ρ, there are five steady states. As ρ moves towards 0.1, the number of steady states drops to three. The damage level for the two lower damage steady states move together as ρ is further increased, and eventually converge past the right side of the panel. When they do, there is only one remaining steady state, which approaches the open access solution. Within the range for ρ shown in the figure, the steady state with the largest instantaneous return always has the lowest damage. For large enough value of ρ, the eigenvalues for the characteristic matrix of the system have nonzero imaginary parts. Thus, we might expect the set of saddle paths converging on the largest instantaneous return steady state to adopt ever increasing oscillations as ρ increases, until for large enough ρ, these saddle paths cease to exist and all paths converge on a point close to the open access solution. For 'reasonable' values of ρ, optimality is consistent with protecting the habitat. The larger ρ is, the more habitat damage it is optimal to tolerate. It is only when ρ becomes very large that the large switch to the open access solution, with its much higher habitat damage level, becomes optimal.

Panel (c) shows the impact of changes in the instantaneous growth rate of the fish stock on the steady states. When the growth rate is zero, the only steady state has zero fish stock and zero habitat damage. The former because the fishery is optimally fished to extinction, and the latter because once effort goes to zero, the habitat must recover. When the growth rate is small but positive, the steady state value of x is close to zero and therefore so too is y. It follows quickly that π_x and π_z must also be close to zero. It has already been shown that when π_x and π_z are zero, the dynamic optimum conditional FOC coincides with the open access solution condition. As $r \to 0$ therefore, assume a form similar to that for the open access solution, excepting around the singularities. Increasing the growth rate works the other way. Ar r grows, the response curve moves towards a monotonic relationship. However, the conditional FOC curves still show strong movement in relation to the singularities for π_z, and multiple steady states remain.

For r, there is an intermediate range where there are three steady states for the dynamic optimum (panel (d)). At larger values of r and a range a bit removed from zero, there are five steady states. When r is very close to zero, there is only one. When there is only one, there is little difference between the open access solution and the optima. The stock grows so slowly that it isn't worth protecting, and once effort falls, the habitat recovers. As r increases, multiple steady states appear, several of which include complex eigenvalues. The open access solution now jumps to a much higher damage level. Further increasing r leads to the static optimum switching to a higher damage steady state, and then for a slightly larger r, the dynamic optimum steady state with the largest instantaneous return also switches to a higher damage level. Within the range explored, the system continues to have five steady states with almost stable damage levels for large values of r. Management of the fishery is now more complex, as optimality does not always coincide with the lowest damage steady state. If stock growth rates are low, then the habitat should be protected. However, for higher stock growth rates, a point is reached where more damage can be tolerated. However, this is not a smooth transition.

Panel (e) shows how the impact of changing q is in some ways opposite to that of changing r. When catchability is high, the effective cost of harvesting is so low that it is not worth retaining stock to support future growth. As for r, high q implies small x and y, which further implies small π_x and π_z. Therefore, the conditional FOC for high q resembles that for low r, which is like the condition identifying the open access solution. Reducing q has a similar effect on the response curve as increasing r. However, it has a very different impact on the shape of the conditional FOC curve. Stock growth influences the conditional FOC through the condition $\dot{x} = 0$ primarily, while catchability enters directly in $\partial H/\partial y = 0$ as well.

The different influence of q is apparent in panel (f), where catchability is varied. For large values of q, there is only one solution. Reducing q leads first to the emergence of one more feasible steady state, and then two more. These additional steady states tend to include complex eigenvalues until q is smaller. Eventually, there is one more feasible steady state that appears, shortly before two disappear. Throughout all of this, the instantaneous return maximizing steady state and the static optimum involve low damage. If catchability is uncertain, or if it is changing, policy should still prevent habitat damage from becoming too large.

Figure 3 plots how steady states respond to changes in parameters that are part of the habitat damage dynamics. Panel (a) shows the impact of changing the habitat recovery rate. When recovery is slow, behaviour is dominated by the damage caused and the inertia of the change. The response function bends back on itself more than for the base case, and the conditional FOC is more extreme in its form. Of note is the fact that for extremely large and small values of z, there is little movement in the conditional FOC curves. There is considerable action between the two levels of z where π_z is undefined, and these levels of z change in response to changes in α and θ. For α, increasing α brings the singularity axes closer together. As it also flattens the response function, bringing down the range of z values where steady states can occur.

[Figure 3 about here.]

Panel (b) shows how changing α has a fairly pronounced influence on the presence or lack thereof of multiple equilibria. Of note is that for all three parameters, over the ranges shown the highest instantaneous return occurs for the lowest damage steady state. Uncertainty about the value of α does not justify accepting a higher level of ecosystem damage. If the habitat recovers slowly, it should still be managed in such a way that the damage is kept low. However, if the habitat is already severely damaged, then it is not so clear what to do. The open access solutions for slow recovery tend to involve high damage. In fact, the IA line for very low levels of α is likely incorrectly placed, as the range of z included in the analysis only extended a small amount past 2. If the habitat has been severely degraded, then the question of optimality depends heavily on the discount rate. If the discount rate is very low, it is always optimal to move to the highest instantaneous return steady state. However, for higher discount rates there will be a cutoff, a Skiba surface, beyond which it is optimal to move to other steady states. For low α, which corresponds to open access solutions with high habitat damage, it is likely that moving to the low damage solution is not optimal.

Panel (c) shows the influence of habitat damage inertia. The impacts generally mirror those of the recovery rate, in that greater inertia makes the response curve bend back on itself more and leads to more radical curves in the conditional FOC functions. Moving to panel (d), it follows that high inertia levels correspond to high damage open access solutions, but at the same time the highest instantaneous return steady states are those with the lowest damage levels. A fishery that is presently at a low damage level should therefore be kept there. However, as for the recovery rate, when inertia is high and open access prevails, it is not clear without more analysis whether or not it is optimal to move back to the low damage steady state.

Finally, panel (e) and (f) show how the system behaves for a range of effort impacts. Increasing impact moves the response function in a similar fashion to increasing catchability. However, the movement of the conditional FOC curves is not as that for catchability, but rather resemble those for changes in the discount rate. Unlike α and θ, m does not enter into the relation defining where π_z is undefined. Thus, that portion of the conditional FOC curves which closely follows the singularity does not move as m is increased. Therefore, the habitat damage for the low damage case approaches a constant level as m increases, as seen in panel (f). In contrast, the open access solution is driven by the greater damage caused by effort, which leads to an increase in damage at the open access solution. As for both recovery and inertia, optimal management of a new fishery likely involves maintaining that fishery at a low habitat damage level. The value of z at which π_z is undefined provides something of a threshold for the value of z, even if the actual value of m is not accurately know. For low values of m it is likely also optimal to return an open access fishery to the dynamic or static optima. However, if m is large, then again the present value of a path from the open access solution to a low damage steady state may be inferior to a path that attains a steady state with a higher damage level.

Finally, figure 4 plots the net surplus, the sum of profit and consumer surplus, and harvest against each of the parameter value ranges explored in figures 2 and 3, for the open access, static optimum, and instantaneous return maximizing dynamic optimum. In all cases, the open access surplus is less than that for the dynamic and static optima. For parameters from the classic model, ρ, r and q, there exists a parameter range where surplus at the dynamic optimum exceeds that of the static optimum. This of course corresponds to solutions where harvest along the dynamic optimum supply curve (panel d of figure 1) lie to the right of the static optimum supply curve. The upward jump of the dynamic optimum seen in panel (d) of figure one is evidenced as a kink in panel (b) of the present figure. For both ρ and q, the gap between the static optimum and dynamic optimum increases as the parameter increases. The discount rate effect is self evident. With high catchability, the value of the stock externality is eliminated. This translates into a larger catch earlier, and lower steady state surplus.

[Figure 4 about here.]

While the static optimum instantaneous return always exceeds that of the dynamic optimum in panels (d) through (f). The steps in the open access reflect two influences. First, the inability to find a solution in some cases shows up as a surplus of zero. Second, around the step to the higher level, some open access solutions may not have been admitted because they would occur for values of z outside the range that was evaluated. These results emphasize the management implications suggested above. If the habitat has dynamics similar to those used in this analysis, it will generally be optimal to manage a new fishery in such a way that the habitat is protected. The gain from doing so, relative to that which would be earned at the open access solution is large. However, if the system has already reached the open access outcome, then it may not be optimal to move it back to a low damage situation.

4 Conclusions

Traditional fishery models largely ignore the state of the habitat in which the fish exist, relying on simple logistic growth. However, several natural systems exhibit more complex behaviour, including multiple stable equilibria. In this paper, shallow lake dynamics have been used to model a habitat damage and recovery process, where fishing effort causes habitat damage. The health of the habitat held no value in itself, but impacted on economic agents through its effect on the carrying capacity of the fish stock. The shallow lake dynamics interact with the logistic dynamics of the fish stock to generate as many as five steady states. For many, but not all, situations the largest period returns accrue to the lowest damage steady state. It is argued that this implies a new fishery should be managed so as to keep the habitat from deteriorating. Uncertainty about the precise nature of the habitat process does not seem to weaken this implication, while variation in the parameters of the classic fishery model itself may. The existence of multiple equilibria and complicated dynamics implies that classic management tools - quotas and taxes on harvest and effort - are unlikely to be effective if a single value is chosen. However, more complex policies, such as a combination of a large lump sum tax on the fishery together with an effort subsidy or a period during which a full or partial moratorium is used, may serve to restore a system that has moved beyond the optimal level of habitat degradation.

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A Classic Fisheries Problem

Let the social planer's objective be

$$\max_y \int_0^\infty \left\{ \int_0^{h=qxy} [A - Bh]\, dh - cy \right\} e^{-\rho t} dt$$

subject to $\dot{x} = rx(1 - x/K) - h$. This assumes a linear demand function and the standard stock effect on harvest. The Hamiltonian for this problem is

$$H = \int_0^{h=qxy} [A - Bh]\, dh - cy + \pi\, [rx(1 - x/K) - h]$$

The first order conditions are

$$\begin{aligned} 0 &= \tfrac{\partial H}{\partial y} = && [A - Bqxy]\, qx - c - \pi qx \\ \rho\pi - \dot{\pi} &= \tfrac{\partial H}{\partial x} = && [A - Bqxy]\, qy + \pi\, [r(1 - 2x/K) - qy] \\ \dot{x} &= \tfrac{\partial H}{\partial \pi} = && rx(1 - x/K) - qxy \end{aligned}$$

One approach to analyze this problem is to take the time derivative of $\partial H/\partial y$, eliminate $\dot{\pi}$, and qualitatively analyze the behaviour in $x \times y$ space. In what follows, I will instead qualitatively analyze the relationship in $x \times \pi$ space, the space of the system state and costate.

Any analysis is going to require the elimination of one of x, y or π. Eliminating π is accomplished using the first order condition, which must hold at all time. However, this also means that we are looking at $\dot{x} = 0$

and $\dot{\pi} = 0$ isoclines in $x \times y$ space. The alternative approach is to examine things in the $x \times \pi$ space. The first order condition can be used, as the downward sloping linear demand curve enables this. However, in higher dimensions this elimination may not be as easy. Thus, an alternative is to create a response curve, where both $\dot{x} = 0$ and $\dot{\pi} = 0$ simultaneously, in $x \times \pi$ space, and a curve mapping where the first order condition is satisfied and at the same time one of the time derivatives is also zero, a conditional FOC. In this case, setting $\dot{x} = 0$ leads to

$$y = r(1 - x/K)/q$$

which can be used to derive expressions for the response curve and the FOC curve.

The response function simplifies to

$$\pi = -\frac{(Br^2x - Ar)K^2 + (Arx - 2Br^2x^2)K + Br^2x^3}{\rho K^2 + Krx}$$

while the first order condition, with $\dot{x} = 0$ imposed, resolves to

$$\pi = -\frac{(Bqrx^2 - Aqx + c)K - Bqrx^3}{Kqx}$$

Notice that for both relationships, if price is constant ($A > 0$, $B = 0$), the higher order polynomial terms in x vanish, and with it some of the complexity of the system. Quick inspection shows that the response function is sensitive to changes in r and ρ, but has no dependence on q or c, while the first order condition expression depends on q, r, and c, but not ρ.

The response functions and first order condition functions for the default case and several alternate parameter values is shown in Figure 5. Response functions slope downward when $x = 0$ and terminate at $\pi = 0$ when $x = K$, while the conditional FOC functions slope upwards, and trend towards $-\infty$ as $x \to 0$ from above. Notice that a high intrinsic growth rate together with the impacts of the downward sloping demand cure leads to a negative shadow value of the dynamic constraint at the steady state. This cannot occur if $B = 0$, as in this case the response curve is strictly greater than zero for $x < K$.

[Figure 5 about here.]

B Algebraic Derivation

With some manipulation, the first order condition can be solved in terms of x and z, with the condition that $\dot{x} = 0$, $\dot{\pi}_x = 0$ and $\dot{\pi}_z = 0$. From $\dot{x} = 0$, a solution for y has already been found. Placing this solution into the steady state condition $\dot{\pi}_x = 0$ yields

$$\pi_x = \frac{r[1 - x(z+1)]\{A - Brx[1 - x(z+1)]\}}{\rho + rx(z+1)}$$

This relation has a singularity where $rx(z+1) = -\rho$. Since this requires x or z to be less than zero, this singularity is not relevant to any feasible solutions.

From the steady state condition $\dot{\pi}_z = 0$, together with $\dot{\pi}_x = 0$ and $\dot{x} = 0$, it follows that

$$\pi_z = \frac{-rx^2\pi_x}{D_z}$$

where

$$D_z(z) = \rho + \alpha - \theta\left(\frac{2\phi^2 z}{(z^2 + \phi^2)^2}\right)$$

a function depending only on z and parameters.

The first order condition for the control variable can be written as

$$0 = \{A - Brx[1 - x(z+1)]\}qx - c - \pi_x(qx + mrx^2/D_z)$$

which can be expanded as a polynomial in x. For this expansion, begin by noticing that the only place x appears in the denominator is in π_x, as part of $\rho + rx(z+1)$. Beginning by expanding the instantaneous return terms $\{A - Brx[1 - x(z+1)]\}qx - c$

$$\begin{aligned}
&= \frac{[Bqr(z+1)x^3 - Bqrx^2 + Aqx - c][\rho + rx(z+1)]}{\rho + rx(z+1)} \\
&= \frac{Bqr^2(z+1)^2x^4 + Bqr(\rho - r)(z+1)x^3 + qr[A(z+1) - B\rho]x^2 + [Aq\rho - cr(z+1)]x - c\rho}{\rho + rx(z+1)}
\end{aligned}$$

Now turning to the portion reflecting future impacts, $\pi_x(qx + mrx^2/D_z)$

$$
\begin{aligned}
&= \frac{r[1-x(z+1)]\{A - Brx[1-x(z+1)]\}}{\rho + rx(z+1)}(qx + mrx^2/D_z) \\
&= \frac{-Br^2(z+1)^2x^3 + 2Br^2(z+1)x^2 - r[rB + A(z+1)]x + Ar}{\rho + rx(z+1)}(qx + mrx^2/D_z) \\
&\quad \left\{-\frac{Bmr^3(z+1)^2}{D_z}x^5 + Br^2(z+1)\left[\frac{2mr}{D_z} - q(z+1)\right]x^4\right. \\
&= -r^2\left[\frac{Bmr + Am(z+1)}{D_z} - 2Bq(z+1)\right]x^3 + r\left[\frac{Amr}{D_z} - Bqr - Aq(z+1)\right]x^2 + Aqrx\left.\vphantom{\frac{1}{1}}\right\} \\
&\quad \times [\rho + rx(z+1)]^{-1}
\end{aligned}
$$

Collecting terms from these two expressions, the implicit conditional FOC is

$$
\begin{aligned}
&\left\{\frac{Bmr^3\left(z+1\right)^2}{D_z}x^5 + 2Br^2\left(z+1\right)\left[q(z+1) - \frac{mr}{D_z}\right]x^4 + r\left[Bq(z+1)(\rho - 3r) + \frac{Bmr^2 + Amr(z+1)}{D_z}\right]x^3\right. \\
&\qquad -r\left[Bq(\rho - r) - 2Aq(z+1) + \frac{mrA}{D_z}\right]x^2 + \left[Aq(\rho - r) - cr(z+1)\right]x - c\rho\left.\vphantom{\frac{1}{1}}\right\} \\
&\qquad \times \left[\rho + rx(z+1)\right]^{-1} = 0
\end{aligned}
$$

With parameter values given, at any value of z, a set of solutions for x can be found.

List of Figures

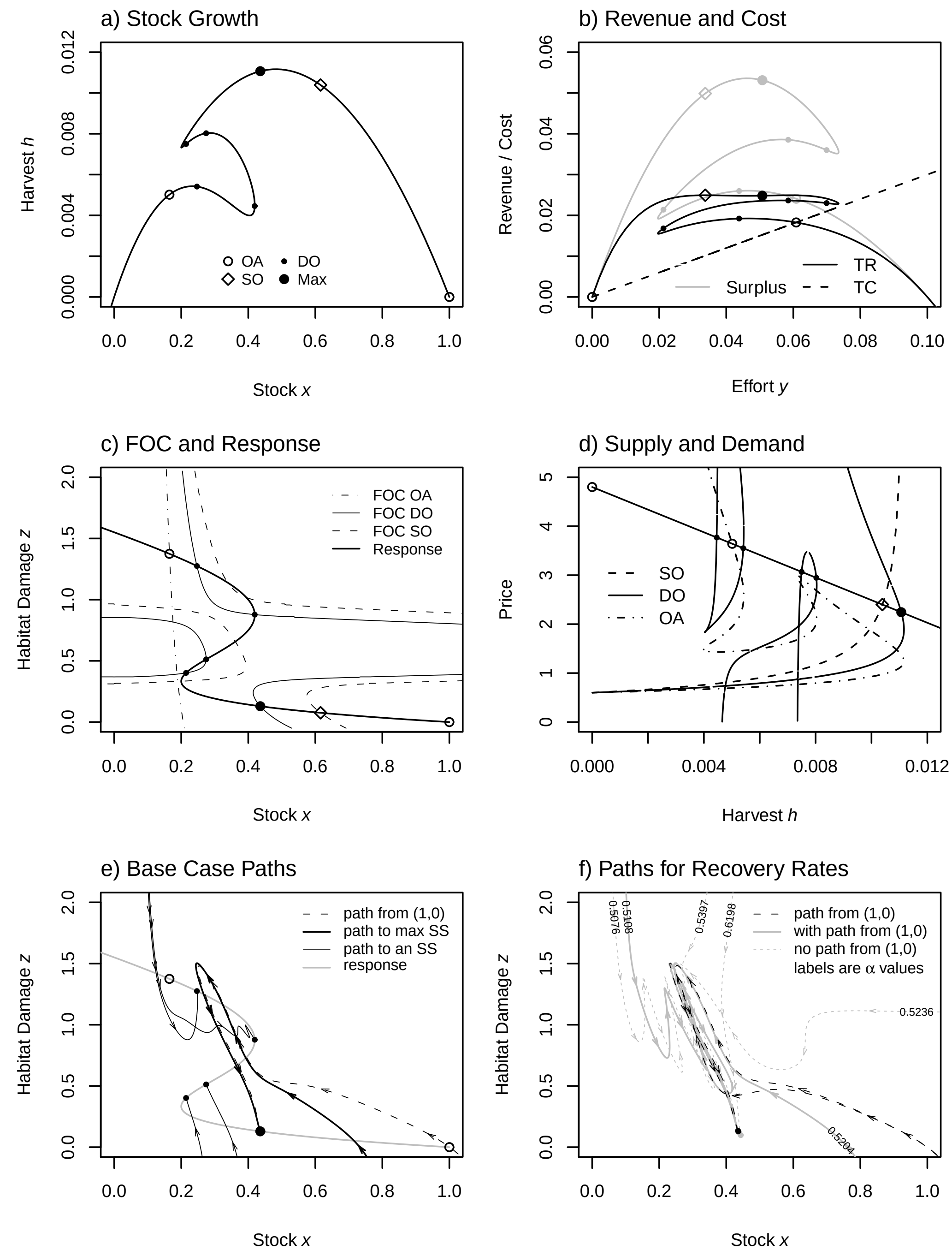

Figure 1: Panels (a) through (e) plot results for the default case, together with the static optimum ($\rho = 0$) and open access ($\rho = \infty$) solutions. Panel (f) plots results for the default case, with variation in α, the rate of habitat recovery.

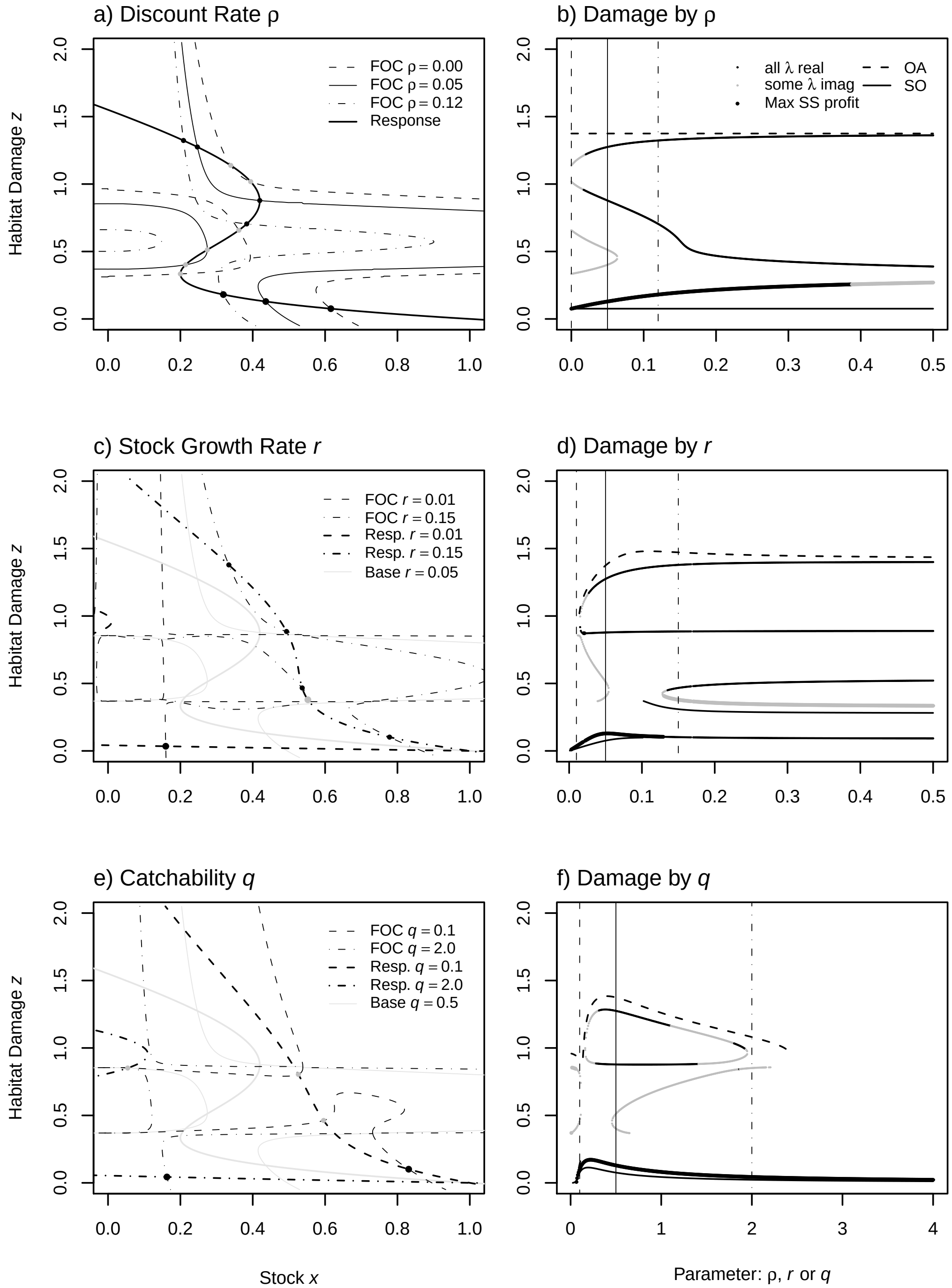

Figure 2: Impact of variation in discount rate (ρ), stock growth rate (r), and catchability (q) on steady states. Left panels plot response function and conditional first order conditions. Right panels plot steady state habitat damage against parameter value.

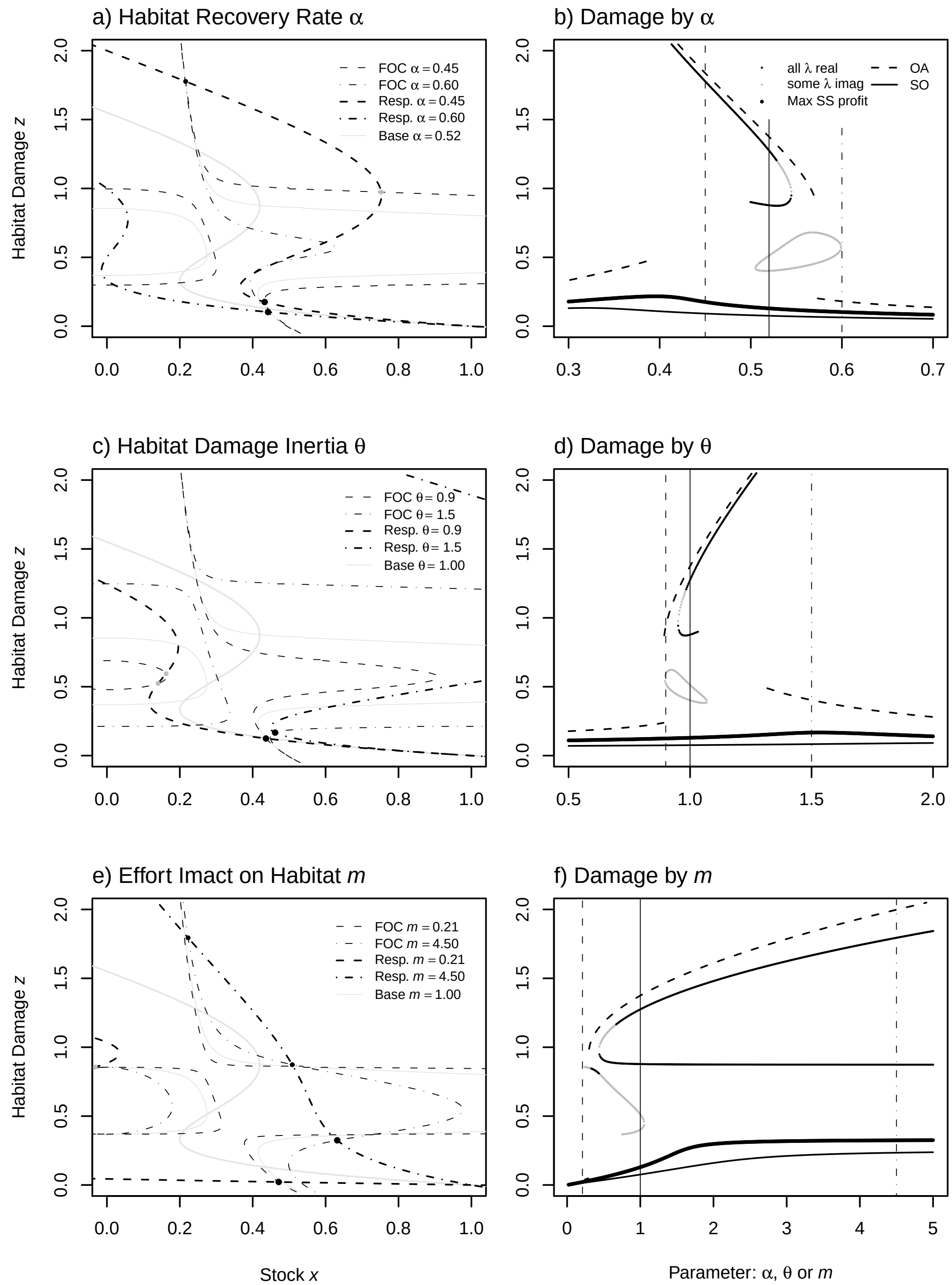

Figure 3: Impact of variation in the habitat recovery rate (α), habitat damage inertia (θ), and effort impact (m) on steady states. Left panels plot response function and conditional first order conditions. Right panels plot steady state habitat damage against parameter value.

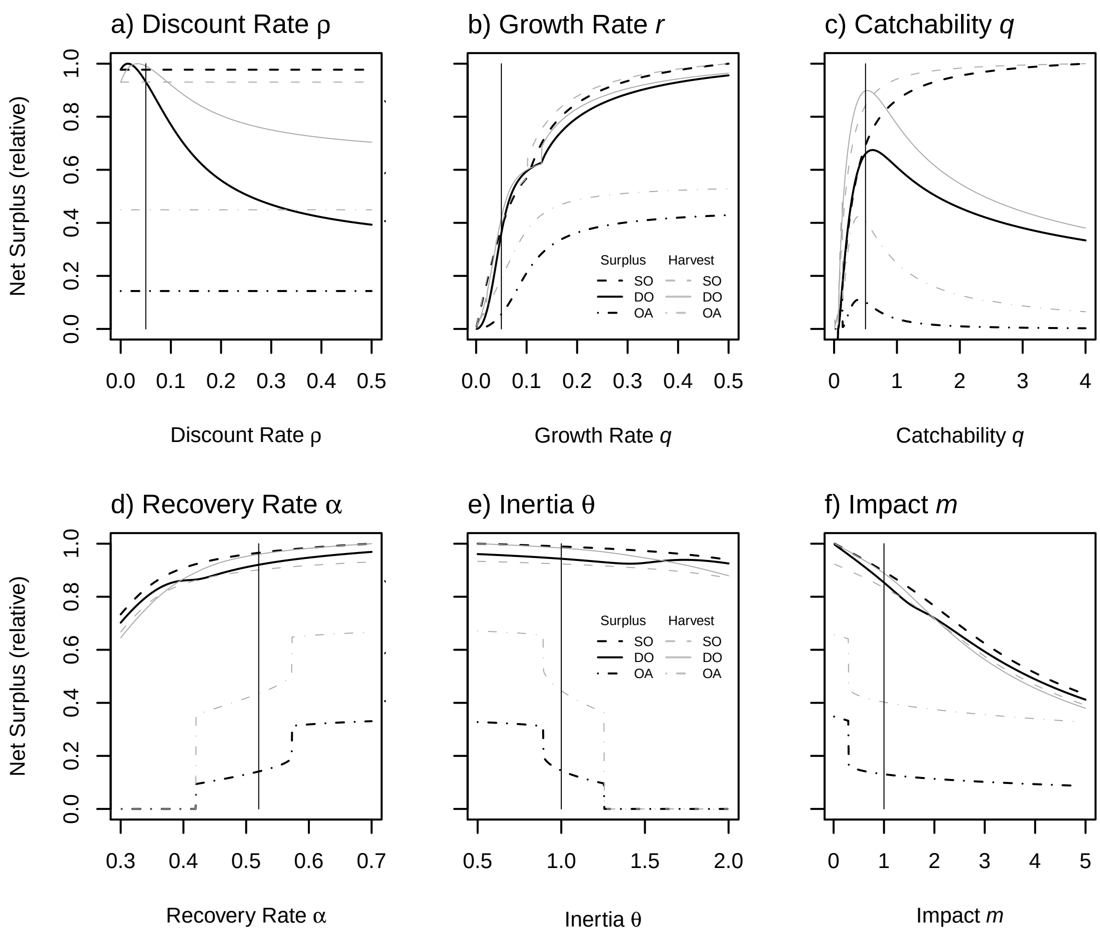

Figure 4: Net surplus and harvest for static optimum (SO), dynamic optimum (DO) and open access (SO) solutions, as a function of parameter variations.

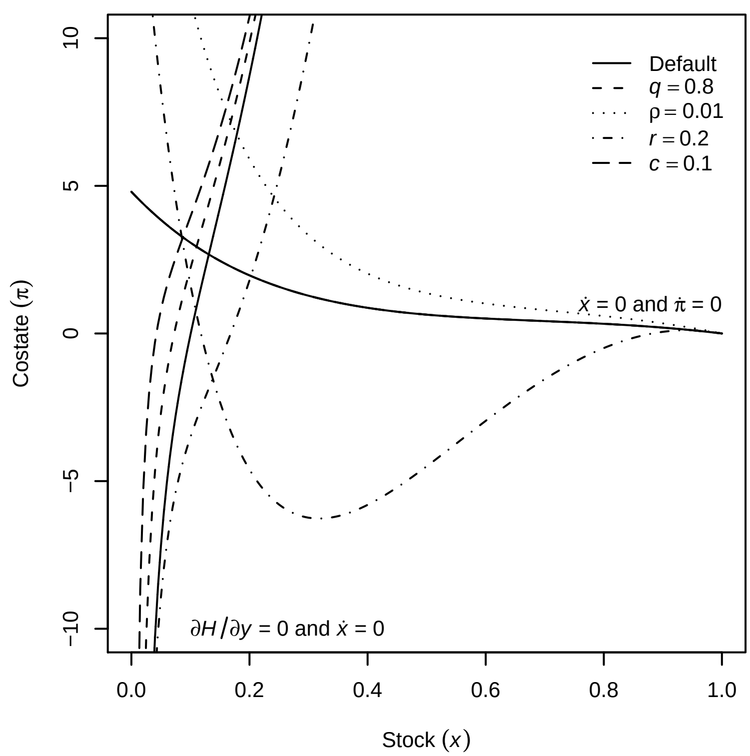

Figure 5: Response functions and conditional first order condition functions. Note that for variation in ρ, the conditional FOC does not move, while for changes in c, response function does not move.

List of Tables

Table 1: Default parameter values.

Stock		Habitat		Economic	
Growth	$r = 0.05$	Impact	$m = 1$	Discount rate	$\rho = 0.05$
Catchability	$q = 0.5$	Recovery	$\alpha = 0.52$	Demand intercept	$A = 4.8$
		Inertia	$\theta = 1$	Demand slope	$B = 231$
		Scale	$\phi = 1$		