Experimental and numerical investigation of terahertz transmission through strongly scattering sub-wavelength size spheres

S. Mujumdar, K. J. Chau, and A. Y. Elezzabi

Ultrafast Photonic and Nano-Optical Laboratory, Department of Electrical and Computer Engineering, University of Alberta, Edmonton T6G 2V4, Canada

(Received 24 August 2004; accepted 25 October 2004)

We report on experimental and numerical studies of terahertz propagation in strongly scattering random media. The experimental variations of the terahertz pulse group delay and scattering-induced effects such as temporal pulse distortion, spectral decay, and power attenuation as a function of sample thickness are in excellent agreement with those predicted from a Monte Carlo photon migration model. The transmitted pulses are analyzed with a classical effective medium approximation. Due to the subwavelength size of the random scatterers, it is found that the effective medium approximation underestimates the accumulated pulse phase acquired by the high frequencies during pulse propagation. © 2004 American Institute of Physics.

The applications of terahertz (THz) radiation are rapidly expanding. In particular, ultrafast THz radiation imaging is emerging as a powerful technique to spatially map a wide variety of objects. THz frequencies possess several notable advantages for imaging. Objects buried within dielectric structures can be imaged using THz radiation due to the transparency of most dielectrics in this regime. Moreover, the nonionizing nature of THz radiation permits noninvasive biological imaging. Unfortunately, the image quality in such applications is inherently influenced by scattering induced by the sample inhomogeneities, reducing both the transmitted power and the spatial resolution. For continued development in THz radiation imaging, a comprehensive understanding of the role of scattering events in THz radiation propagation is vital.

Despite the significant influence of scattering on THz radiation propagation, experimental and theoretical work has been limited. Recent research by Pearce et al. has examined off-axis scattered light where the sample length, L, and transport mean free path, l, were chosen such that L/l < 40. Earlier, a study of on-axis THz transmission was reported by the same authors through thin slabs of monodisperse Teflon spheres. In order to study the weakly scattering regime, L/l was selected to be less than 10. In more complex systems, however, both weak and strong scattering are present. In the context of on-axis THz transmission through a random medium, this range of scattering is unexplored.

In this letter, we measure and describe through simulation the scattering effects on THz propagation through dispersively and strongly scattering random media where L/l can exceed 300. In contrast to the weakly scattering regime, where scattering-induced effects only cause distortions of the spectral phase, significant temporal pulse reshaping is anticipated in a strongly scattering regime. Through examination of this effect, in addition to transmitted THz pulse features such as group delay, pulse width, spectral width, and the transmitted power, we have fully characterized THz transmission through a strongly scattering medium. In conjunction with experimental measurements, a numerical Monte Carlo model based on time-dependent photon transport is implemented. The calculated variations of the THz pulse group propagation delay and scattering-induced effects, such as temporal pulse distortion, spectral decay, and power attenuation as a function of sample thickness are in excellent agreement with the experimental data. For the strongly scattering sample used, the high sensitivity of the experimental pulse width to the effective dispersion of the medium provides ideal conditions for verifying the applicability of the widely accepted effective medium approximation (EMA). It is found that for scattering size parameter x ≳ 1, the EMA underestimates the accumulated pulse phase acquired during pulse propagation. Such insight is vital to the eventual application of THz radiation to image subwavelength size, highly scattering structures.

To measure the THz radiation transmission, we employ the setup described in Ref. 6. A collimated THz pulse is directed towards a 5 cm diameter sample cell containing polydisperse, subwavelength size alumina spheres with mean diameter of 100 μm. Owing to the large bandwidth of the THz pulse and the coherent detection employed, variation of the sample cell width from 0 to 14 mm permits the investigation of all regimes of scattering (0.001 < L/l* < 300). To numerically describe THz pulse propagation through the medium, the incident pulse is first modeled in the frequency domain as an ensemble of THz photons that mark the straight-line trajectories of the radiation inside the sample. The amplitude spectrum of the incident pulse, \( E_{\text{in}}(\omega) \), is decomposed into constituent frequencies with a resolution of 30 GHz, and the profile of \( E_{\text{in}}(\omega) \) is used to construct a cumulative probability distribution of the frequency of the photons. The mean free path \( \ell(\omega) \) of the radiation and the anisotropy parameter \( g(\omega) \) are calculated using Mie theory, whereas the dispersion properties of alumina are taken from Ref. 8. The sample boundaries are defined by the planes \( x = 0 \text{ mm}, x = 50 \text{ mm}, y = 0 \text{ mm}, y = 50 \text{ mm}, z = 0 \text{ mm}, \) and \( z = L \text{ mm} \). A photon at a given frequency \( \omega \) is launched at \((x_0, y_0, z_0)=(25,25,0)\) mm into the sample where it undergoes a three-dimensional random walk. The polar and azimuthal angular coordinates of the photon at the time of launch are chosen to be \((\theta_i, \phi_i) = (0,0)\), so as to simulate a collimated beam. At the \( i \)th step of the random walk, the photon undergoes a linear displacement \( p_i \) and an angular displacement \((\theta_i, \phi_i)\), giving a new position \((x_j, y_j, z_j)\).
+p_1 \sin \theta_1 \sin \phi_1 + p_2 \sin \theta_2 \cos \phi_2 + p_3 \cos \theta_3). For an exponential distribution with a mean free path \( \ell \), the path lengths are determined as \( p_i = -\ell \ln \Sigma \) where \( \Sigma \) is a variate uniformly distributed over \([0,1]\). The azimuthal angle \( \phi_i \) is uniformly distributed whereas the polar angle \( \theta_i \) is chosen from a Henyey–Greenstein distribution \( P(\theta_i) = (1 - g^2)/4 \pi (1 + g^2 - 2g \cos \theta_i)^{3/2} \). The trajectories of \( 10^9 \) photons determine the transport of the pulse. Using the two-component Bruggeman effective medium approximation, \( E \) the phase of a transmitted photon is calculated from its path length. The number of transmitted photons as a function of frequency generates the transmitted electric field spectral amplitude, \( E(\omega) \), while the spectral phase, \( \Phi(\omega) \), is derived from the cumulative phase of all transmitted photons at frequency \( \omega \).

The data shown in Fig. 1(a) demonstrates the effect of the sample thickness on the transmission spectrum. In contrast to the low frequency constituents, which exhibit longer \( \ell \), the high frequency components of the incident pulse decay rapidly as the sample thickness increases. As a result, the bandwidth of the pulse spectrum decreases markedly from 0.97 to 0.5 THz over \( 0 < L < 4 \) mm. As shown in Fig. 1(b), this spectral narrowing is predicted by the photon migration model. For samples thicker than 4 mm, the higher frequencies (1 THz) have already been scattered outside of the detection acceptance angle, and only a gradual decrease in the spectral width is observed. Over the entire scattering range, the measured and calculated spectral bandwidths agree well as shown in Fig. 2(a). The excellent agreement underlines the applicability of the model to predict spectral quantities of on-axis field propagation. Figure 2(b) depicts the measured and calculated normalized power transmitted through the scattering samples. Similar to the bandwidth profile, a rapid fall (~60\%) in the THz transmission power is witnessed for \( 0 < L < 6 \) mm. The marked power decay in this thickness interval is due to selective scattering of the high frequency components. In this critical region, where the transmitted power is most sensitive to the sample thickness, an excellent agreement in the calculated and measured transmission power is observed. For sample thickness greater than 6 mm both the experimental and calculated transmission powers are weakly dependent upon the sample width. A slight offset between the experiment and calculations in this region is caused by the polarization sensitivity of the experimental detection, which is nonexistent in the model. Overall, the model succeeds in emulating the observed bandwidth and power of the THz transmission as a function of \( L \).

Figure 3 depicts the calculated and experimentally measured THz pulses through the random medium \( L=1.68 \) and 7.3 mm. While the pulse widths from the experiment and theory differ, the group delays exhibit good quantitative agreement for thickness values up to 14 mm, as shown in Fig. 4. Due to strong scattering, the experimental pulse shape develops increasingly large subsidiary peaks following an initial bipolar pulse as the sample thickness increases. It is interesting to note that these distinct pulse shape features are well described by the model. To investigate the origin of this effect, Fig. 3(b) compares two numerical pulse profiles, where one is generated from the ballistic photons and the other from the ballistic and the earliest arriving scattered photons. Evidently, the trailing oscillations in the pulse are caused by the scattered photons reaching the detector.

While the model and experiment agree in regards to group delay, scattering-induced pulse distortion, spectral decay, and power attenuation, it should be noted that this agreement is based on the phase insensitivity of the said characteristics. Clearly, the pulse width is highly sensitive to the effective dispersion of the random medium and the relative phase accumulated during propagation. Thus, comparison of the calculated pulse widths with the experiment offers a good test into the applicability of the EMA commonly used to describe electromagnetic wave propagation through random media. By mapping the random sample on an effective homogeneous medium described by the Bruggemann EMA, \( \sigma \) it is found that the calculated pulse widths underestimate the experimental pulse duration by a factor of \( \sim 2 \). This discrepancy confirms the inadequacy of classical EMA for spherical scatterers with a size parameter above \( x \sim 1 \). Accordingly, for higher frequencies of the THz pulse, phase modifications are required. As there exists no EMA that yields accurate...
results for $x > 1$, we empirically modified the EMA by introducing a small second order corrective phase factor. This was done as follows: we expanded the unmodified phase into a series expansion in terms of $v$ yielding $F'_{\text{tr}}(v) = A v + 0.01 v^2$ where $A$ is the constant of proportionality. We then iteratively modified the coefficient of the second order term, while testing the agreement between the experimental and numerical pulses. This was done until a good fit was obtained, from which a phase of $\tilde{F}_{\text{tr}}(v) = A v + 0.09 v^2$ was achieved. A plot of the modified and unmodified phases in the inset of Fig. 5 illustrates the phase correction introduced by this method. The calculated pulse shape maintains the pulse delay, and the pulse stretch now agrees excellently with the experimental pulse. The modified phase offers insight into the behavior of radiation in random media where the size parameter is beyond the applicability range of the EMA.

In summary, we report on experimental and theoretical studies of THz propagation in highly scattering random media. Pulse delay and large scattering-induced modulation in both the temporal and spectral domain have been observed and accurately described by a photon migration model. This investigation is useful in a wide range of applications related to THz transmission through heterogeneous materials exhibiting strong scattering effects.

This work is supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

7. C. Bohren and D. Huffman, Absorption and Scattering of Light by Small Particles (Wiley, New York, 1983).