Analysis of cholera epidemics with bacterial growth and spatial movement

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Overview

1. Introduction

2. The Model and The Reproduction Number $R_0$

3. Deriving $R_0^{ODE}$, $R_0^{PDE}$ and the stability of S.S.

4. Numerical and Graphical Analysis
Introduction

During 2010-2012,

More than 530,000 reported cases, over 7000 deaths,
During 2008-2009,

Nearly **100,000** reported cases, over **4000** deaths,
Cholera is an acute intestinal infectious disease caused by the bacterium *Vibrio cholerae*. It can spread rapidly and lead to death within days if left untreated. The complexity of cholera dynamics stems from the fact that both routes are involved:

1. direct transmission (human-to-human)
2. indirect transmission (environment-to-human)

In this presentation, we implement new models that incorporate both intrinsic bacterial dynamics and spatial variation.
We will be using the standard SIR Model with the inclusion of the intrinsic bacterial dynamics.
The Model and The Reproduction Number $R_0$

\[
\begin{align*}
\frac{dS}{dt} &= b - Sf_1(I) - Sf_2(B) - dS + \sigma R, \\
\frac{dI}{dt} &= Sf_1(I) + Sf_2(B) - (d + \gamma)I, \\
\frac{dR}{dt} &= \gamma I - (d + \sigma)R, \\
\frac{dB}{dt} &= \xi I + h(B) - \delta B.
\end{align*}
\]
The Model and The Reproduction Number $R_0$
If $R_0 > 1$, $\ldots$
The Model and The Reproduction Number $R_0$

Uncontrollable Spread!!

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If $R_0 < 1$, 


The Model and The Reproduction Number $R_0$
Deriving $R_0^{\text{ODE}}, R_0^{\text{PDE}}$ and the stability of S.S.
Assume:

- \( f_1(I) = \beta_h I \) and \( f_2(B) = \beta_e B / (B + \kappa) \), where \( \beta_h, \beta_e \) represent the direct and indirect transmission parameters respectively, and \( \kappa \) is the concentration of V.cholera in the environment.

- The intrinsic bacterial growth rate: \( h(B) = gB(1 - B / K) \), where \( g = h'(0) \) and \( K \) is the maximal capacity of free-living bacteria in the environment.
Observe that $g = h'(0)$ and $\xi$ play a vital role in shaping $R_0$.

- Suppose $g < \delta$, we can see the transition between $R_0 < 1$ to $R_0 > 1$ under these assumptions:
  1. $g$ is fixed at a constant value in the range and $\xi$ increases to cross $\Gamma$
  2. $\xi$ is fixed and $g$ increases to cross $\Gamma$

- Suppose $g > \delta$, it is clear that the shedding rate $\xi$ becomes less important.
Numerical and Graphical Analysis

Cholera Epidemics

March 9, 2016 19 / 25
This confirms our last result on \( \lim_{D_2, D_4 \to \infty} R_0^{PDE} = R_0^{ODE} \).
For the fixed \( g \) and \( \xi \), \( R_0^{PDE} \) increases and becomes uniformly greater than one for all the possible values of \( g > 0 \) and \( \xi > 0 \).
If $R_0 < 1,$
If $R_0 > 1$,,
Thank you! :)