Math 350 **Winter 2016**

Review # 2

1. (a) Give the definition of the Principal branch Log(z) of the logarithm.

(b) Describe the domain and range of Log(z)? Where is Log(z) analytic?

(c) Is it true that Log(x^2)=2 Log(z) for all $z \in \mathbb{C}$?

(d) Sketch and describe carefully the domain of analyticity of Log($2z-1$).

2. Express the following quantities in the $u+iv$ form.

(a) $\sinh(1+\pi i)$

(b) $\mathcal{L}_{\pi/2}(-\sqrt{3}+i)$

3. Find all values of z for which $\text{Log}(z^2+1) = \frac{i\pi}{2}$.

4. Find all values of z for which $e^{z^2} = 1$.

5. Evaluate

(a) $i^{\sqrt{2}}$ (b) $\left(\dfrac{2i}{1+i}\right)^{1/3}$ (c) $(\sqrt{3}+i)^{1+i}$

6. Compute by two different methods:

$$\int_i^{2i} (z^2 - 2e^{2z})\, dz.$$

7. Let γ be the boundary of the circle of radius 2 centered at the origin. Compute

(a) $\displaystyle\int_\gamma \frac{1}{z^2+1}\, dz$ (b) $\displaystyle\int_\gamma \frac{z+i}{z-3}\, dz$ (c) $\displaystyle\int_\gamma \frac{\cos z}{z(z^2-1)}\, dz$

8. Let z_0 denote a fixed complex number, and let γ be a simple closed contour with positive orientation such that z_0 lies in the interior of γ. Derive the following formula:

$$\oint_\gamma \frac{dz}{(z-z_0)^n} = \begin{cases} 0 & n \neq 1, \\ 2\pi i & n = 1. \end{cases}$$

9. (a) State Cauchy's Integral Theorem.

(b) State the Deformation Invariance Theorem

(c) Show that Cauchy's Integral Theorem implies the Deformation Invariance Theorem.

10. Prove (2) $\Leftrightarrow$ (3) in the Path Independence Lemma.