

## Practice Test #1

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Show your work. Where appropriate, your solutions should include explanations and references to theorems.

1.(a) Evaluate  $(\sqrt{3} - i)^7$ .

(b) Find all distinct values of  $i^{1/2}$ .

2. Describe the set of points  $z$  in the complex plane that satisfy the following equations, and determine which of these is a domain.

(a)  $| \operatorname{Re}(z + 3 + i) | > 1$

(b)  $\operatorname{Im}(z) < \operatorname{Re}(z)$

3. Verify that the function  $u(x, y) = e^x \sin y$  is harmonic, and find the harmonic conjugate of  $u$ .

4. Use the rigorous definition of limits to prove that

$$\lim_{z \rightarrow 1+i} (6z - 4) = 2 + 6i.$$

5. Show that  $\overline{\cos(iz)} = \cos(\overline{iz})$ .

6. Answer any ONE of the following questions:

(i) Use the definition of the derivative to show that  $f(z) = \bar{z}$  is nowhere differentiable.

(ii) Prove that if  $f(z)$  is differentiable at  $z_0$ , then  $f(z)$  is continuous at  $z_0$ .

(iii) Explain why an analytic function satisfies the Cauchy Riemann equations. Your answer should include a derivation of the Cauchy Riemann equations.