UNIVERSITEIT GENT

BIOSTAT
Department of Applied Mathematics, Biometrics and Process Control

A spatio-temporal statespace model for river network data

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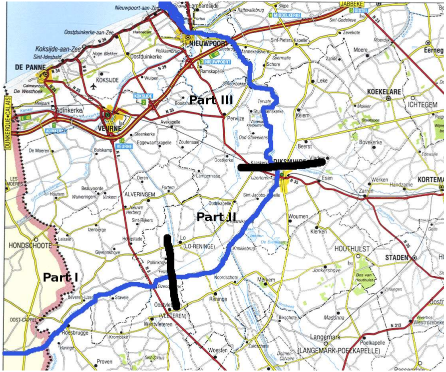
Introduction

- Important legislation concerning water quality (WQ): Water Framework Directive (WFD)
- Major goal: maintain and improve the aquatic environment
- In Flanders: VMM $\rightarrow$ develop basin management plans
- Detect impact of previous actions. Assess improvement/trend in WQ

The Yzer Catchment



1. Yzer
   1. length 76 km, 44 km in Belgium
   2. area 1101 km^2
   3. mouth Nieuwpoort: complex of sluices.
   4. Eutrophication due to nutrient pollution: high nitrate

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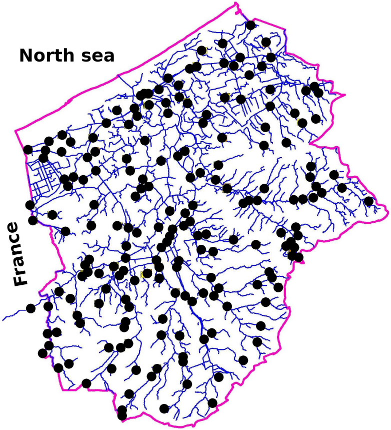

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   1. Sampling location 1: dependence in time

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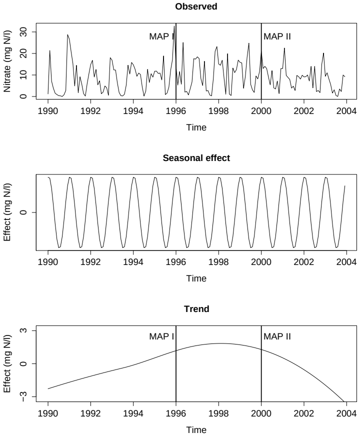

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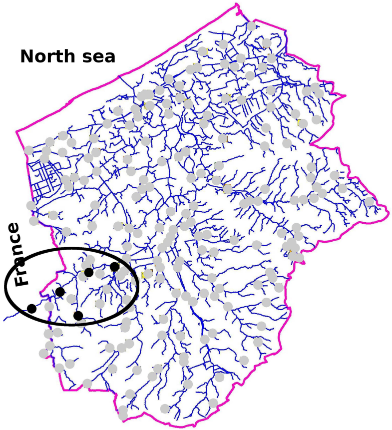

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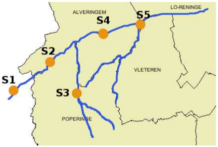

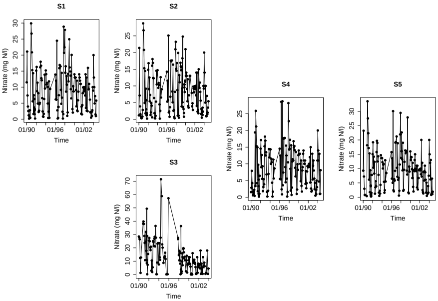

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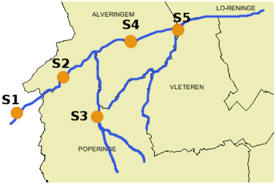
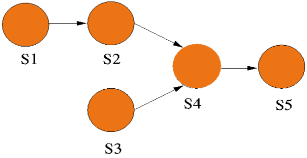
Methods: Spatio-Temporal model

1. Spatial dependence structure
2. Spatio-temporal dependence structure
3. Observation model

Spatial dependence structure (SDP)



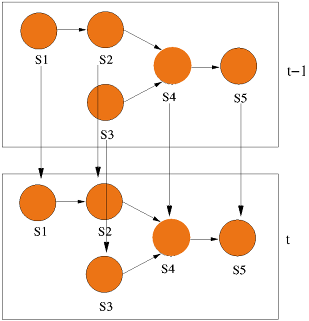


$$\mathbf{S} = \mathbf{AS} + \gamma$$

- River networks:
  - The water flows in 1 direction $\rightarrow$ Causal interpretation of correlations
  - Unidirectional correlation structure
- Isolated river: Directed Acyclic Graph $\rightarrow$ conditional independence
- $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \rho_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{24} & \rho_{34} & 0 & 0 \\ 0 & 0 & 0 & \rho_{45} & 0 \end{bmatrix}$
- $\gamma \sim MVN(\mathbf{0}, \mathbf{\Sigma}_\gamma)$, $\mathbf{\Sigma}_\gamma$ diag

Spatio-temporal dependence structure

$$\mathbf{S}_t = \mathbf{A}\mathbf{S}_t + \mathbf{B}\mathbf{S}_{t-1} + \eta_t$$

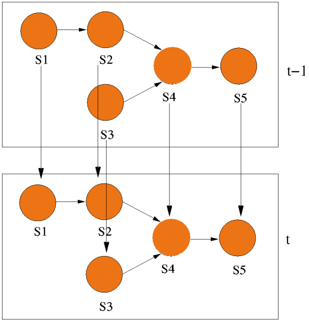


- Unidirectional dependence in time
- Assumption: observation at t only depends on observation at $t-1 \longrightarrow$ AR(1) process
- **B** Diagonal matrix with AR coef (only monthly observations)
- $\eta_t \sim MVN(\mathbf{0}, \mathbf{\Sigma}_\eta)$ ($\mathbf{\Sigma}_\eta$ diag)

Spatio-temporal dependence structure

$$\mathbf{S}_t = \mathbf{A}\mathbf{S}_t + \mathbf{B}\mathbf{S}_{t-1} + \boldsymbol{\eta}_t$$

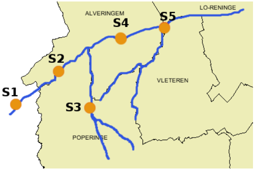
$$\mathbf{S}_t = \boldsymbol{\Phi}\mathbf{S}_{t-1} + \boldsymbol{\delta}_t$$



- Unidirectional dependence in time
- Assumption: observation at t only depends on observation at $t-1 \rightarrow$ AR(1) process
- $\mathbf{B}$ Diagonal matrix with AR coef (only monthly observations)
- $\boldsymbol{\eta}_t \sim MVN(\mathbf{0}, \boldsymbol{\Sigma}_\eta)$ ($\boldsymbol{\Sigma}_\eta$ diag)
- $\boldsymbol{\Phi} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$
- $\boldsymbol{\delta}_t \sim MVN(\mathbf{0}, \mathbf{Q})$, $\mathbf{Q} = (\mathbf{I} - \mathbf{A})^{-1}\boldsymbol{\Sigma}_\eta(\mathbf{I} - \mathbf{A})^{-T}$

Observation Model

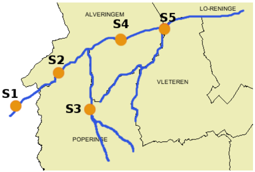
$$\begin{cases} \mathbf{S}_t = \mathbf{\Phi}\mathbf{S}_{t-1} + \delta_t \\ \mathbf{Y}_t = \mathbf{S}_t + \epsilon_t \end{cases}$$



- Model of S only holds for isolated river model
- Reality: disturbance
  → put **S** in observation model
- Error term: $\epsilon_t \sim MVN(\mathbf{0}, \mathbf{\Sigma}_\epsilon)$
  → spatial correlation due to disturbances
- Up to now only covariance structure modelled

Observation Model

$$\begin{cases} \mathbf{S}_t = \mathbf{\Phi}\mathbf{S}_{t-1} + \delta_t \\ \mathbf{Y}_t = \mathbf{X_t}\beta + \mathbf{S}_t + \epsilon_t \end{cases}$$



- Model of S only holds for isolated river model
- Reality: disturbance
  $\rightarrow$ put **S** in observation model
- Error term: $\epsilon_t \sim MVN(\mathbf{0}, \mathbf{\Sigma}_\epsilon)$
  $\rightarrow$ spatial correlation due to disturbances
- Up to now only covariance structure modelled
- Mean model:
  $\text{E}\left\{\mathbf{Y}_t\right\} = \mathbf{X}_t\beta$

Parameter Estimation

- Kalman Filter and Kalman Smoother
- ECM algorithm
  - E step: Calculate expected log likelihood of State Space model
  - CM step1: Parameters dependence structure
  - CM step 2: Parameters of the mean model

Kalman Filter

State Space model: likelihood can be factorized using the Kalman Filter

$$
\begin{array}{rcccl}
E[S_t|t-1] & = & \mathbf{a}_{t|t-1} & = & \mathbf{\Phi a}_{t-1} \\
E[(\mathbf{S}_t - \mathbf{a}_{t|t-1})(\mathbf{S}_t - \mathbf{a}_{t|t-1})^T] & = & \mathbf{P}_{t|t-1} & = & \mathbf{\Phi P}_{t-1}\mathbf{\Phi}^T + \mathbf{Q} \\
E[S_t] & = & \mathbf{a}_t & = & \mathbf{a}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{F}_t^{-1}\mathbf{v_t} \\
E[(\mathbf{S}_t - \mathbf{a}_t)(\mathbf{S}_t - \mathbf{a}_t)^T] & = & \mathbf{P}_t & = & \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{F}_t^{-1}\mathbf{P}_{t|t-1} \\
E[v_t v_t^T] & = & \mathbf{F}_t & = & \mathbf{P}_{t|t-1} + \mathbf{\Sigma}_\epsilon
\end{array}
$$

with innovation $\mathbf{v_t} = (\mathbf{Y}_t - \mathbf{a}_{t|t-1} - \mathbf{X}_t\beta)$

$$
\log L_{\mathbf{Y}} = \sum_{t=1}^{N} \log L_{\mathbf{y_t}|\mathbf{Y_{t-1}}} \sim -\frac{1}{2}\sum_{t=1}^{N} \log|\mathbf{F}_t| - \frac{1}{2}\sum_{t=1}^{N} \mathbf{v_t^T}\mathbf{F_t}^{-1}\mathbf{v_t}
$$

Kalman Smoother

- Smoothed estimates $\mathbf{a}_{t|N}$ for $\mathbf{S_t}$ conditionally on all N
  For $t = N-1, \ldots, 0$

$$\mathbf{a}_{t|N} = \mathbf{a}_t + \mathbf{P}_t^*(\mathbf{a}_{t+1|N} - \mathbf{a}_{t+1|t})$$
$$\mathbf{P}_{t|N} = \mathbf{P}_t + \mathbf{P}_t^*(\mathbf{P}_{t+1|N} - \mathbf{P}_{t+1|t})\mathbf{P}_t^{*T}$$
$$\mathbf{P}_t^* = \mathbf{P}_t\Phi^T\mathbf{P}_{t+1|t}^{-1}$$

- Lag-one covariance smoothers (Digalakis, Rohlick and Osendorf 1993)
  $\Rightarrow$ Forward recursion:

$$\mathbf{P}_{t,t-1|t} = (\mathbf{I} - \mathbf{P}_{t|t-1}\mathbf{F}_t^{-1})\boldsymbol{\Phi}\mathbf{P}_{t-1}$$

  $\Rightarrow$ Backward recursion:

$$\mathbf{P}_{t,t-1|N} = \mathbf{P}_{t,t-1|t} + (\mathbf{P}_{t|N} - \mathbf{P}_t)\mathbf{P}_t^{-1}\mathbf{P}_{t,t-1|t}$$

Using the Kalman filter for GLS

- Apply same Kalman filter to $\mathbf{Y_t}$ and each of the columns of $\mathbf{X_t}$

⇒ A $p \times 1$ vector of innovations, $\mathbf{Y_t^*}$ on $\mathbf{Y_t}$ and

⇒ a $p \times m$ matrix of innovations, $\mathbf{X_t^*}$ on $(\mathbf{X_{1t}}, \ldots, \mathbf{X_{mt}})$ are produced.

⇒ Run recursions for $\mathbf{P_{t|t-1}}$, $\mathbf{P_{t|t}}$ and $\mathbf{F_t}$ only once rather than $m+1$ times.

- GLS estimator of β becomes

$$\hat{\beta}_{\mathbf{GLS}} = \left[\sum_{t=1}^{N} \mathbf{X_t^{*T} F_t^{-1} X_t^*}\right]^{-1} \sum_{t=1}^{N} \mathbf{X_t^{*T} F_t^{-1} y_t^*}. \tag{1}$$

- $\mathbf{v_t}$ become $\mathbf{v_t} = \mathbf{y_t^*} - \mathbf{x_t^*}\hat{\beta}_{\mathbf{GLS}}$.
- Maximisation possible by classical numerical algorithms
- Here ECM algorithm

Existing EM algorithms

1. $l_c(\mathbf{\Psi}) = \log L_{Y_N,S_N}(\mathbf{\Psi})$ the joint log-likelihood of $\mathbf{Y}_N$ and $\mathbf{S}_N$.
2. Problem: unobservable state process $\mathbf{S}$
3. Use an EM algorithm (e.g. Shumway and Stoffer 1982)
4. RMN topology: restrictions on parametrisation

$\Rightarrow$ $\mathbf{Q}$ and $\mathbf{\Phi}$ have some parameters in common

$\Rightarrow$ Adapt existing EM algorithm for SS models

5. Presence of the exogenous variables $\Rightarrow$ use ECM algorithm.

$\Rightarrow$ Split M-step in two CM-steps.

$\Rightarrow$ CM step 1: Update the parameters of the dependence structure $\mathbf{\Psi}_{\alpha}$ given the current values of the mean model β.

$\Rightarrow$ CM step 2: estimate of β using the updated values of $\mathbf{\Psi}_{\alpha}$.

ECM algorithm

1. Choose initial estimates: $\boldsymbol{\Psi}^0$
2. **E-step**: Calculate $Q(\boldsymbol{\Psi}, \boldsymbol{\Psi}_\alpha^k, \beta^k) = \mathrm{E}\left\{l_c(\boldsymbol{\Psi})|\mathbf{Y}_N, \boldsymbol{\Psi}^k\right\}$
3. **CM-step 1**: Find the covariance parameters $\boldsymbol{\Psi}_\alpha^{k+1}$ that maximise $Q(\boldsymbol{\Psi}, \boldsymbol{\Psi}_\alpha^k, \beta^k)$
4. **CM-step 2**: Find β^{k+1} that maximises $Q(\boldsymbol{\Psi}, \boldsymbol{\Psi}_\alpha^{k+1}, \beta^k)$
5. Repeat steps 2-4 until convergence

E-step

$$Q(\boldsymbol{\Psi},\boldsymbol{\Psi}^k) \sim -\frac{1}{2}\,\mathrm{E}\left\{\log|\Sigma_{S_0}| + \mathbf{S}_0^T\Sigma_{S_0}^{-1}\mathbf{S}_0|\mathbf{Y},\boldsymbol{\Psi}^k,\beta^k\right\}$$
$$-\frac{1}{2}\,\mathrm{E}\left\{N\log|\Sigma_\eta| + \sum_{t=1}^N(\mathbf{S}_t-\mathbf{A}\mathbf{S}_t-\mathbf{B}\mathbf{S}_{t-1})^T\Sigma_\eta^{-1}(\mathbf{S}_t-\mathbf{A}\mathbf{S}_t-\mathbf{B}\mathbf{S}_{t-1})|\ldots\right\}$$
$$-\frac{1}{2}\,\mathrm{E}\left\{N\log|\Sigma_\epsilon| + \sum_{t=1}^N(\mathbf{Y}_t-\mathbf{X}\beta-\mathbf{S}_t)^T\Sigma_\epsilon^{-1}(\mathbf{Y}_t-\mathbf{X}\beta-\mathbf{S}_t)|\ldots\right\}.$$

Exponential family $\Rightarrow$ sufficient statistics $|\mathbf{Y}$

$$E[\mathbf{S}_t|\mathbf{Y}] = \mathbf{a}_{t|N}$$
$$E[\mathbf{S}_t\mathbf{S}_t|\mathbf{Y}] = \mathbf{P}_{t|N} + \mathbf{a}_{t|N}\mathbf{a}_{t|N}^T$$
$$E[\mathbf{S}_t\mathbf{S}_{t-1}|\mathbf{Y}] = \mathbf{P}_{t,t-1|N} + \mathbf{a}_{t|N}\mathbf{a}_{t-1|N}^T.$$

Maximise the obtained expected likelihood in the CM steps

CM-step 1: Update parameters of dependence structure given β^k

$$
\begin{aligned}
Q(\boldsymbol{\Psi}, \boldsymbol{\Psi}^k) \sim & -\frac{1}{2}\,\mathrm{E}\left\{\log|\Sigma_{S_0}| + \mathbf{S}_0^T \Sigma_{S_0}^{-1} \mathbf{S}_0 | \mathbf{Y}, \boldsymbol{\Psi}^k, \beta^k\right\} \\
& -\frac{1}{2}\,\mathrm{E}\left\{N \log|\Sigma_\eta| + \sum_{t=1}^{N} (\mathbf{S}_t - \mathbf{A}\mathbf{S}_t - \mathbf{B}\mathbf{S}_{t-1})^T \Sigma_\eta^{-1} (\mathbf{S}_t - \mathbf{A}\mathbf{S}_t - \mathbf{B}\mathbf{S}_{t-1}) | \ldots\right\} \\
& -\frac{1}{2}\,\mathrm{E}\left\{N \log|\Sigma_\epsilon| + \sum_{t=1}^{N} (\mathbf{Y}_t - \mathbf{X}\beta - \mathbf{S}_t)^T \Sigma_\epsilon^{-1} (\mathbf{Y}_t - \mathbf{X}\beta - \mathbf{S}_t) | \ldots\right\}.
\end{aligned}
$$

$$\hat{\boldsymbol{\Sigma}}_{S_0} = \mathbf{P}_{0|N}$$

Replace all sufficient statistics by their conditional expectations

CM-step 1: Update parameters of dependence structure given β^k

$$Q(\mathbf{\Psi}, \mathbf{\Psi}^k) \sim -\frac{1}{2}\,\mathrm{E}\left\{\log|\Sigma_{S_0}| + \mathbf{S}_0^T\Sigma_{S_0}^{-1}\mathbf{S}_0|\mathbf{Y}, \mathbf{\Psi}^k, \beta^{\mathbf{k}}\right\}$$
$$-\frac{1}{2}\,\mathrm{E}\left\{N\log|\Sigma_\eta| + \sum_{t=1}^{N}(\mathbf{S}_t - \mathbf{A}\mathbf{S}_t - \mathbf{B}\mathbf{S}_{t-1})^T\Sigma_\eta^{-1}(\mathbf{S}_t - \mathbf{A}\mathbf{S}_t - \mathbf{B}\mathbf{S}_{t-1})|\ldots\right\}$$
$$-\frac{1}{2}\,\mathrm{E}\left\{N\log|\Sigma_\epsilon| + \sum_{t=1}^{N}(\mathbf{Y}_t - \mathbf{X}\beta - \mathbf{S}_t)^T\Sigma_\epsilon^{-1}(\mathbf{Y}_t - \mathbf{X}\beta - \mathbf{S}_t)|\ldots\right\}.$$

$$\mathrm{E}\left\{\log L_{\mathbf{S}}(\mathbf{\Psi})|\ldots\right\} \sim -\tfrac{1}{2}\textstyle\sum_{i=1}^{p}\mathrm{E}\left\{N\log\Delta\sigma_{\eta_i} + \frac{1}{\sigma^2_{\eta_i}}\sum_{t=1}^{N}(S_t^i - \mathbf{A}^{[i]}\mathbf{S}_t^{[i]} - \mathbf{B}^i S_{t-1}^i)^2|\ldots\right\}$$

CM-step 1: Update parameters of dependence structure given β^k

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\begin{aligned}
Q(\mathbf{\Psi}, \mathbf{\Psi}^k) \sim & -\frac{1}{2}\,\mathrm{E}\left\{\log|\Sigma_{S_0}| + \mathbf{S}_0^T \Sigma_{S_0}^{-1} \mathbf{S}_0 | \mathbf{Y}, \mathbf{\Psi}^k, \beta^{\mathbf{k}}\right\} \\
& -\frac{1}{2}\,\mathrm{E}\left\{N \log|\Sigma_\eta| + \sum_{t=1}^{N} (\mathbf{S}_t - \mathbf{A}\mathbf{S}_t - \mathbf{B}\mathbf{S}_{t-1})^T \Sigma_\eta^{-1} (\mathbf{S}_t - \mathbf{A}\mathbf{S}_t - \mathbf{B}\mathbf{S}_{t-1}) | \ldots \right\} \\
& -\frac{1}{2}\,\mathrm{E}\left\{N \log|\Sigma_\epsilon| + \sum_{t=1}^{N} (\mathbf{Y}_t - \mathbf{X}\beta - \mathbf{S}_t)^T \Sigma_\epsilon^{-1} (\mathbf{Y}_t - \mathbf{X}\beta - \mathbf{S}_t) | \ldots \right\}.
\end{aligned}
$$

$$
\left\{
\begin{array}{l}
B_i^{k+1} = \dfrac{\sum_{t=1}^N s_t^i s_{t-1}^i - \left(\sum_{t=1}^N s_t^i \mathbf{s}_t^{[i]\,T}\right)\left(\sum_{t=1}^N \mathbf{s}_t^{[i]} \mathbf{s}_t^{[i]\,T}\right)^{-1}\left(\sum_{t=1}^N s_{t-1}^i \mathbf{s}_t^{[i]}\right)}{\sum_{t=1}^N {s_{t-1}^i}^2 - \left(\sum_{t=1}^N s_{t-1}^i \mathbf{s}_t^{[i]\,T}\right)\left(\sum_{t=1}^N \mathbf{s}_t^{[i]} \mathbf{s}_t^{[i]\,T}\right)^{-1}\left(\sum_{t=1}^N s_{t-1}^i \mathbf{s}_t^{[i]}\right)} \\
\mathbf{A}_i^{k+1} = \sum_{t=1}^N \left[\left(s_t^i - B_i^{k+1} s_{t-1}^i\right) \mathbf{s}_t^{[i]\,T}\right] \left(\sum_{t=1}^N \mathbf{s}_t^{[i]} \mathbf{s}_t^{[i]\,T}\right)^{-1} \\
{\sigma_{\eta_i}^2}^{\,k+1} = \dfrac{\mathrm{RSS}_i^{k+1}}{N}
\end{array}
\right.
$$

Replace all sufficient statistics by their conditional expectations

CM-step 1: Update parameters of dependence structure given β^k

$$
\begin{aligned}
Q(\boldsymbol{\Psi}, \boldsymbol{\Psi}^k) \sim & -\frac{1}{2}\,\mathrm{E}\left\{\log|\Sigma_{S_0}| + \mathbf{S}_0^T \Sigma_{S_0}^{-1}\mathbf{S}_0 | \mathbf{Y}, \boldsymbol{\Psi}^k, \boldsymbol{\beta}^k\right\} \\
& -\frac{1}{2}\,\mathrm{E}\left\{N\log|\Sigma_\eta| + \sum_{t=1}^{N}(\mathbf{S}_t - \mathbf{A}\mathbf{S}_t - \mathbf{B}\mathbf{S}_{t-1})^T \Sigma_\eta^{-1}(\mathbf{S}_t - \mathbf{A}\mathbf{S}_t - \mathbf{B}\mathbf{S}_{t-1})|\ldots\right\} \\
& -\frac{1}{2}\,\mathrm{E}\left\{N\log|\Sigma_\epsilon| + \sum_{t=1}^{N}(\mathbf{Y}_t - \mathbf{X}\boldsymbol{\beta} - \mathbf{S}_t)^T \Sigma_\epsilon^{-1}(\mathbf{Y}_t - \mathbf{X}\boldsymbol{\beta} - \mathbf{S}_t)|\ldots\right\}.
\end{aligned}
$$

$$
\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}{}^{k+1} = \frac{\sum_{t=1}^N(\mathbf{Y}'_t\mathbf{Y}'^T_t) - \sum_{t=1}^N(\mathbf{Y}'_t\mathbf{S}^T_t) - \sum_{t=1}^N(\mathbf{S}_t\mathbf{Y}'^T_t) + \sum_{t=1}^N(\mathbf{S}_t\mathbf{S}^T_t)}{N}
$$

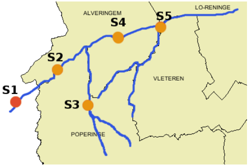
with $\mathbf{Y}'_t = \mathbf{Y}_t - \mathbf{X_t}\boldsymbol{\beta}^{\mathbf{k}}$

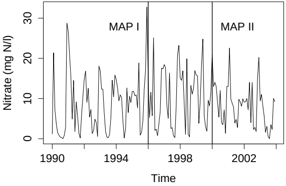
Replace all sufficient statistics by their conditional expectations

CM-step 2: Update β^k given $\mathbf{\Psi}_\alpha^{k+1}$

- $\boldsymbol{\beta}_{k+1}$: FGLS using Kalman Filter with $\mathbf{\Psi}_\alpha^{k+1}$.
- Kalman filter is already available for next E step.

Case Study



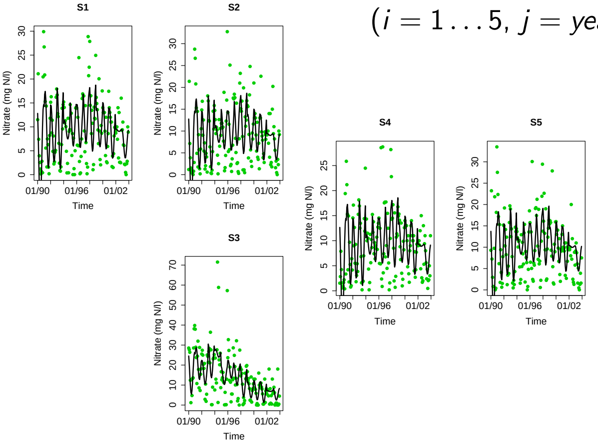


- Has the water quality improved?
- Mean model: ST + annual effect
- Is mean of 2003 different from
  1. the general mean
  2. the mean of 2001-2002

Model for average NO_3^- at S_i on time t

$$E\{y_{i,t}\} = \mu + \alpha_i + \beta_j + (\kappa)_j \mathrm{I}(i) + \gamma_1 \sin(2\pi t/12) + \gamma_2 \cos(2\pi t/12) + (\lambda)_{j,1} \sin(2\pi t/12) + (\lambda)_{j,2} \cos(2\pi t/12)$$

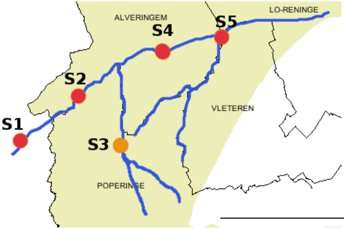
$$(i = 1 \ldots 5,\ j = year_1, \ldots, year_{14})$$



Statistical tests

- $\hat{\beta}(\mathbf{\Psi}_{\alpha}) \xrightarrow{d} MVN(\beta, (\mathbf{X}^T \mathbf{\Sigma}_{Y_N}^{-1} \mathbf{X})^{-1})$
- $\hat{\mathbf{\Sigma}}_{\beta} = (\mathbf{X}^T \mathbf{\Sigma}_{\mathbf{Y_N}}^{-1} (\hat{\mathbf{\Psi}}_{\alpha}) \mathbf{X})^{-1}.$
- A general linear hypothesis to compare the means: $\mathbf{H}\beta = \mathbf{0}$, where $\mathbf{H}$ is the $r \times q$ hypothesis matrix.
- Test: $T = (\mathbf{H}\hat{\beta})^T (\mathbf{H}\hat{\mathbf{\Sigma}}_{\beta}\mathbf{H}^T)^{-1} (\mathbf{H}\hat{\beta})$

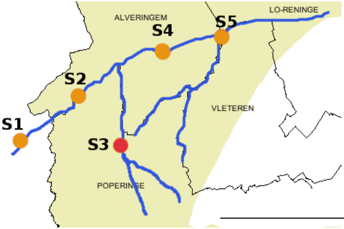
Statistical tests



- H_0: Mean 2003 equal to Mean 2001-2002
- H_1: Mean 2003 different from Mean 2001-2002

Test	contrast	p-value
	Main river ("Regional")	
2003 ↔ 2001-2002	-2.54	0.016
2003 ↔ general mean	-2.88	0.0005
	Joining creek (S3)	
2003 ↔ 2001-2002	-1.32	0.56
2003 ↔ general mean	-8.40	< 0.0001

Statistical tests



- H_0: Mean 2003 equal to Mean 2001-2002
- H_1: Mean 2003 different from Mean 2001-2002

Test	contrast	p-value
	Main river ("Regional")	
2003 ↔ 2001-2002	-2.54	0.016
2003 ↔ general mean	-2.88	0.0005
	Joining creek (S3)	
2003 ↔ 2001-2002	-1.32	0.56
2003 ↔ general mean	-8.40	< 0.0001

References

1. Clement, L. and Thas, O. (2007). Estimating and modelling spatio-temporal correlation structures for river monitoring networks. Journal of Agricultural, Biological, and Environmental Statistics, 12(2), 161-176.
2. Clement, L., Thas, O., Vanrolleghem, P.A. and Ottoy, J.P. (2006). Spatio-temporal statistical models for river monitoring networks. Water, Science and Technology, 53, 9-15.