Searching of Periodic Components in the Lake Mina, Minnesota Time Series

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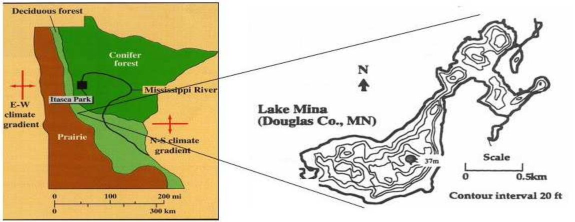
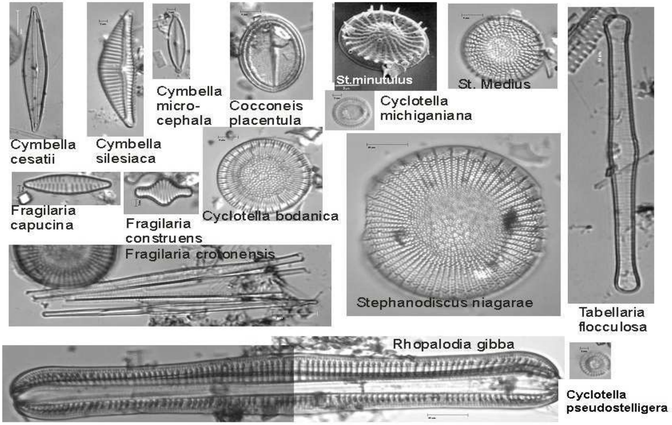
Lake Mina

Lake Mina, Minnesota is located just south of the coniferous/deciduous forest ecotone



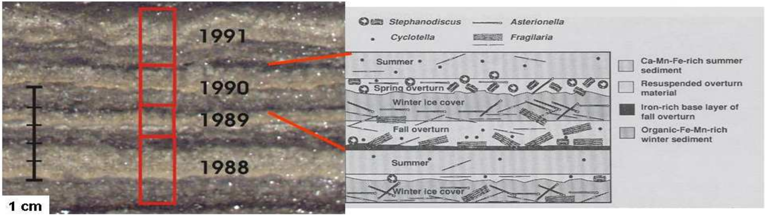
Lake Mina Diatoms

Diatoms are (generally) unicellular microscopic algae with cell walls made of silica.



Varved Core

An example of a Varved Lake Core.



Lake Mina Core

The core taken from Lake Mina

What are we looking for?

- We are looking for periodic components in the historical Diatom and Pollen grain record that can be explained by outside forcing.
- Specifically we are looking for evidence of solar and lunar forcing
- For example
  - 11-year solar cycle
  - 18.6-year lunar cycle
  - 35-year Brückner solar cycle [3]
  - Evidence of the 52-month ENSO cycle.

Multitaper

- We will use the multitaper spectrum estimates[2]
- Multitaper spectral estimates, in there crude form, are averaged direct spectral estimators using Discrete Prolate Spheroidal Sequences (dpss's) of different orders as windowing functions.
- For each taper one computes the *eigencoefficients*,

$$y_k(f) = \sum_{t=0}^{N-1} x(t) v_t^{(k)}(N, W) \exp(-i2\pi f t). \tag{1}$$

The tapers are normalized such that

$$\sum_{t=0}^{N-1} [v_t^{(k)}(N, W)]^2 = 1 \tag{2}$$

Multitaper

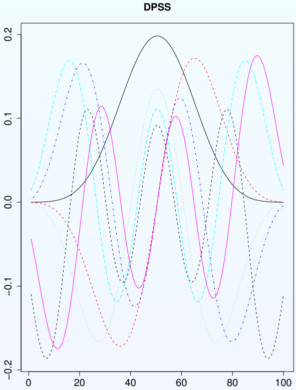


Figure 1: DPSS

Multitaper

- In practice the crude multitaper estimate is not used, instead weights are used to replace averaging the eigencoefficients (1).
- The weights are calculated iteratively from

$$d_k(f) \approx \frac{\sqrt{\lambda_k} S(f)}{\lambda_k S(f) + B_k(f)}, \tag{3}$$

  where $B_k(f)$ is an estimate of the power in the broad-band bias terms.
- The initial estimate and bound for the bias term is given by

$$B_k(f) \leq \sigma^2(1 - \lambda_k). \tag{4}$$

- We also take the multitaper over sectioned overlapped blocks of data and combind these blocks using the arithmetic mean

F-test

The multitaper method allows for F-tests test for periodic components in coloured noise. The harmonic F-test is defined,

$$F(f) = \frac{(K-1)|\hat{\mu}|^2 \sum_{k=0}^{K-1} U_k(N, W; 0)^2}{\sum_{k=0}^{K-1} |y_k(f) - \hat{\mu}(f) U_k(N, W; 0)|^2}, \tag{5}$$

with 2, and $2K - 2$ degrees of freedom, where our mean, $\hat{\mu}(f)$ is estimated by

$$\hat{\mu}(f) = \frac{\sum_{k=0}^{K-1} U_k(N, W; 0) y_k(f)}{\sum_{k=0}^{K-1} U_k^2(N, W; 0)}. \tag{6}$$

Singular Values

- We take the singular value decomposition of a matrices formed by the eigencoefficients, (1), at each frequency, of most prevalent diatoms by the pollen or diatom samples. $\mathbf{Y}(f)$ is $k \times p$

$$\mathbf{Y}(f) = \mathbf{U}(f)\mathbf{\Sigma}(f)\mathbf{V}^{\dagger}(f), \tag{7}$$

- In the general the right singular vectors of $1/\sqrt{n-1}\mathbf{X}$ are the eigenvectors of the covariance matrix $\mathbf{X}^T\mathbf{X}$, where $\mathbf{X}$ is a $n \times p$ data matrix
- The multitaper eigencoefficients were obtained using a time-bandwidth parameter of 3.5, and 9 data tapers

Singular Values

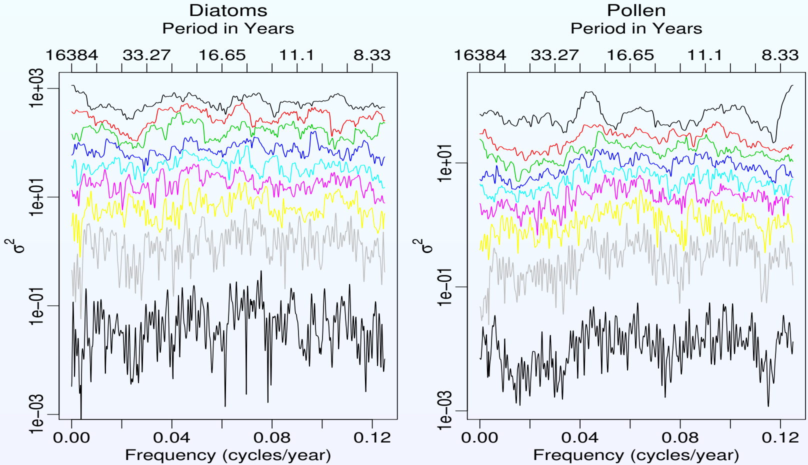


Figure 2: SVD of eigencoefficients and Data

Left Singular Vectors

- We look at the left singular vectors from the decomposition. We conduct a harmonic F-test, (5) of these singular vectors
- The left singular vectors are eigenvectors of the outer product matrix $\mathbf{M}(f) = \mathbf{Y}(f)\mathbf{Y}(f)^{\dagger}$
- Each column of $\mathbf{M}$, contains

$$\mathbf{m_1}(f) = \sum_{i=1}^{p} \mathbf{y_i}(f)\, y_i^{(0)}(f), \tag{8}$$

- The left singular vectors with the largest singular values correspond to dimensions in the data with the most variation at a particular frequency

Left Singular Vector

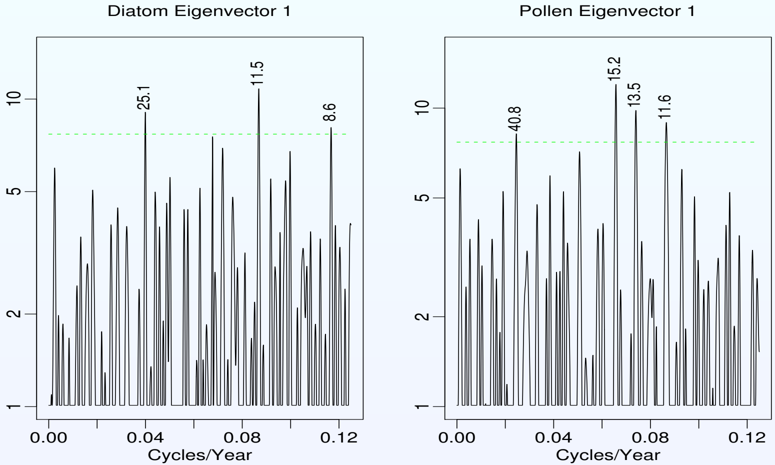
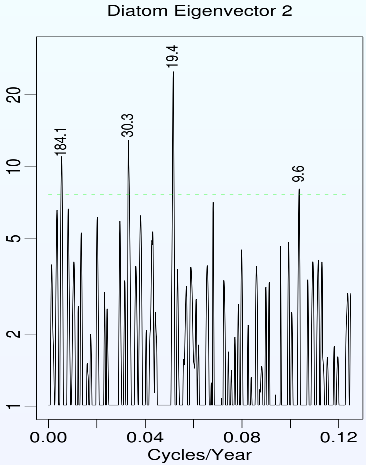


Figure 3: Ftest of Left Eigenvector 1

Left Singular Vector



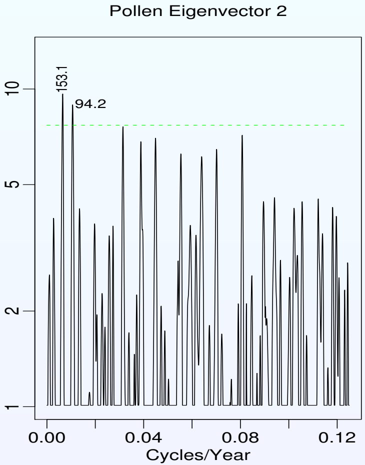


Figure 4: Ftest of Left Eigenvector 2

Significant Peaks 99.5%

SV	Data	Significant Periods	Accountable
1	Pollen	40.76 15.21 13.52 **11.55**	1
1	Diatom	25.05 **11.51** **8.56**	2
2	Pollen	153.12 94.16	0
2	Diatom	184.09 30.28 **19.39** 9.65	1
3	Pollen	455.11 26.47 **19.14** 14.71 12.22	1
3	Diatom	66.06 48.05 37.15	0
		12.59 12.21 11.77	
		10.14 8.21	
4	Pollen	78.39 64.25 **38.73**	3
		29.10 27.26 **17.52**	
		15.88 **10.42**	
4	Diatom	58.94 29.26 21.42 14.98	0

Table 1: Location of Significant Peaks

Canonical Coherence

- Canonical coherence, which is similar to canonical correlation, using multitaper techniques are relative new
- They show the linear relationship between two multivariate sets of time series
- We form the canonical coherence, by taking the SVD of the cross product of left eigenvectors from the original SVD described above

Canonical Coherence

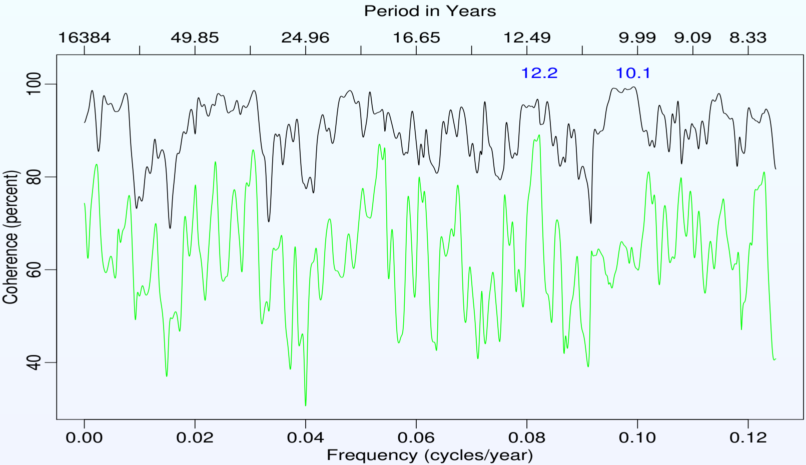
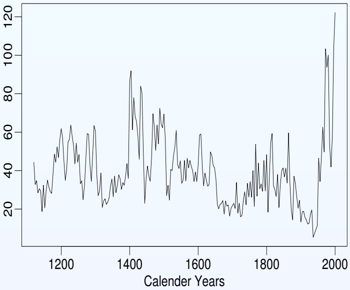


Figure 5: Canonical Coherence

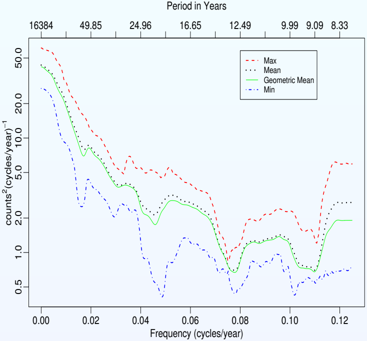
Individual Taxa

- Quercus comprises over one quarter of the pollen sample
- It is a fire-sensitive deciduous tree taxa that is found both dryer mid-Holocene pollen assemblage and colder and moister assemblages [4]
- Fragilaria Crotonensis comprises 35% of the diatom Sample
- This Diatom taxa generally peaks twice a year, once in the late spring and a second time in the Autumn
- The time-bandwidth parameter was set to 3.5, and 7 tapers were used in the Bock Multitaper
- There are 11 blocks and about 84% overlap

Quercus



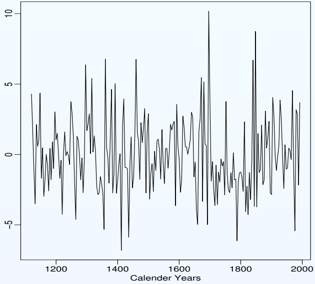
(a) Quercus Time Series



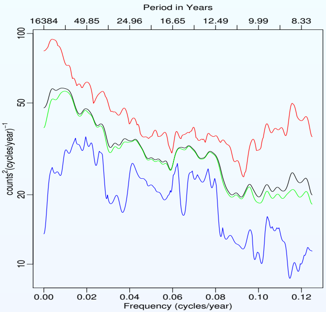
(b) Quercus Spectrum

Figure 6: Figure 6(a) shows initial the Quercus time series, and figure 6(b) plots the block multitaper spectrum.

Fragilaria Crotonensis



(a) Fragilaria Crotonensis Time Series



(b) Fragilaria Crotonensis Spectrum

Figure 7: Figure 7(a) shows initial the Fragilaria Crotonensis time series, and figure 7(b) plots the block multitaper spectrum.

Quercus Spectrogram

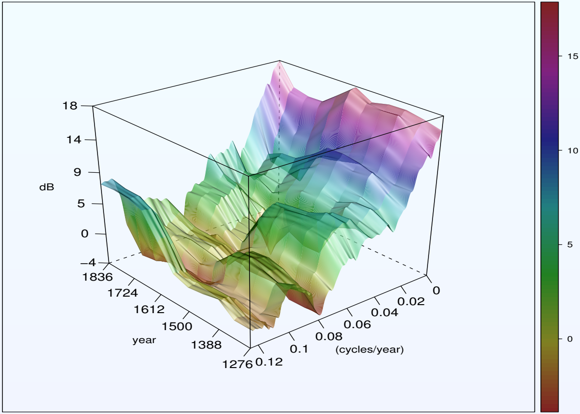


Figure 8: Multitaper Spectrogram of Quercus

Fragilaria Crotonensis Spectrogram

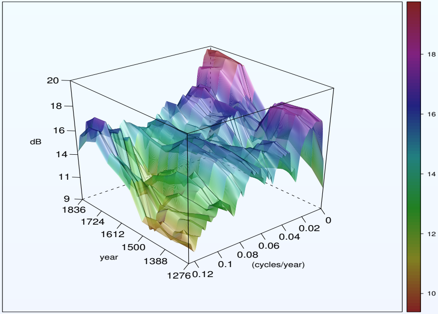


Figure 9: Multitaper Spectrogram of Fragilaria Crotonensis

Block Values for F-test

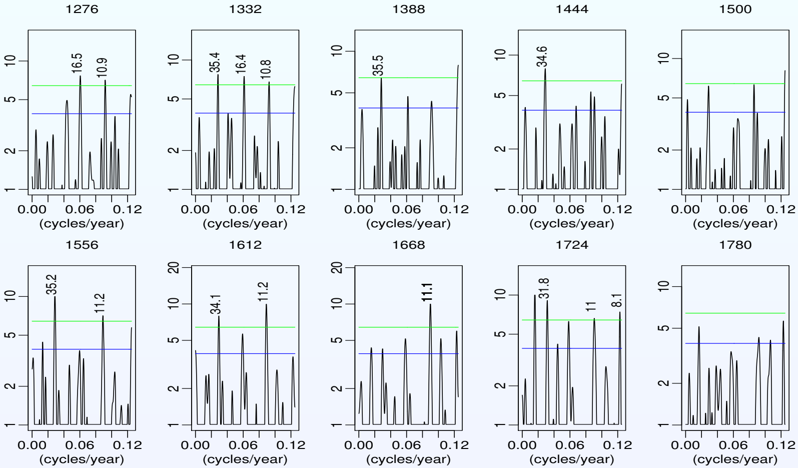


Figure 10: Quercus F-tests for each time block

Moving Values for F-test

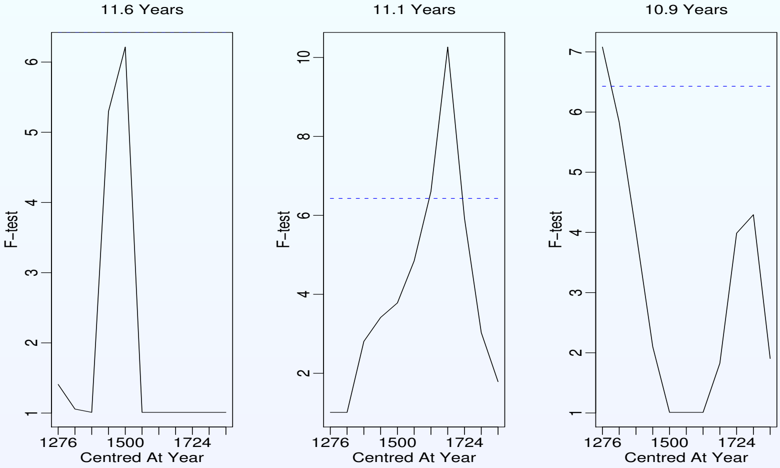


Figure 11: Quercus F-test for fixed frequency

Moving Values for F-test

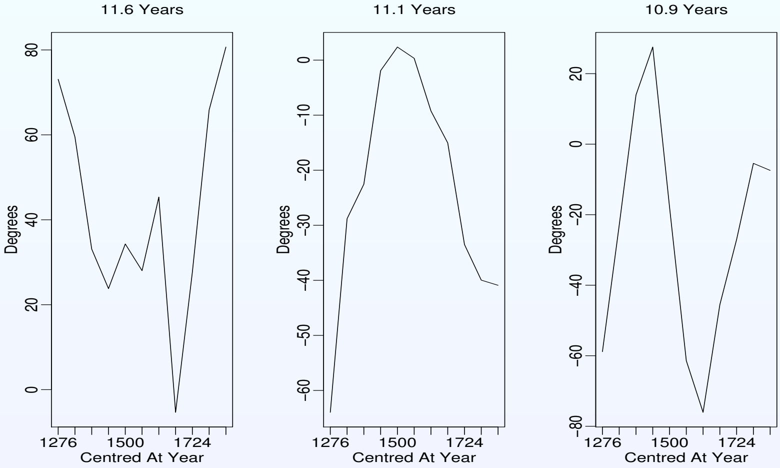


Figure 12: Quercus F-test phase for fixed frequency

Comparing with a Reference Series

- We can compare coherence with a reference set as a way to help rule out aliases
- A reference series of annual tree ring growth from Campito Mountain was obtained [1].
- We needed four-year resolution so we filtered the reference series by projecting the data onto the spaced spanned by the orthonormal dpss's
- We form the projection filter

$$\tilde{\mathbf{x}} = \mathbf{V}\mathbf{V}^T\mathbf{x}. \tag{9}$$

Filter Using Expansion in dpss's

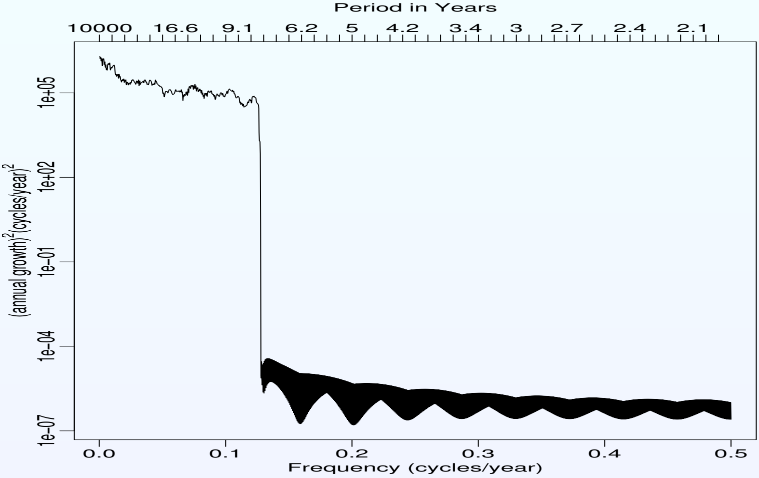


Figure 13: Filtered Campito

Coherence with Reference Series

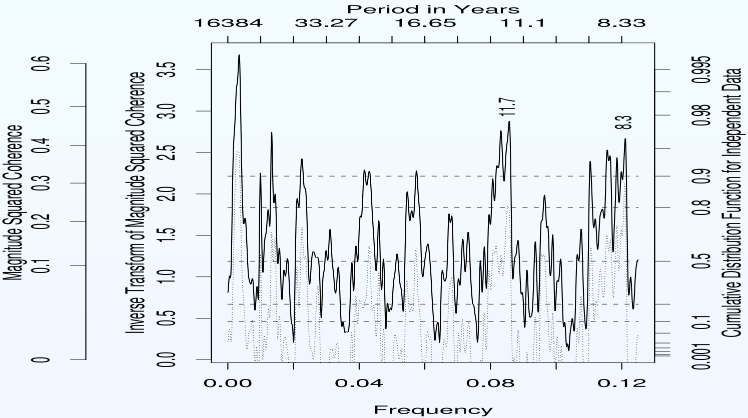


Figure 14: Coherence with Campito Reference Series

Concluding Remarks

- We have found several period line components in the data
- We see evidence of the 11-year and 35-year solar cycle in the two most prevalent of the diatoms and the pollen grains
- In Quercus we see evidence of an 11=year and 35-year period
- We also see evidence of a 16-year period which is not as easily explained
- There are several highly significant period components in the data

Future Work

- Check for evidence of counting errors
- Fit a distribution to the possibility of counting errors and test
- Check for coherence with other reference sets.
- Methods to present, pictorially, information/results of analysis from multiple species.

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Thanks

Thank you